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On the Global Stability Analysis of Corona Virus Disease (COVID-19) Mathematical Model

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In this present work, we investigated the Global Stability Analysis of Corona virus disease model formulated by Atokolo et al in [11]. The COVID-19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic that is ravaging the whole world. By constructing a Lyapunov function, we investigated the stability of the model Endemic Equilibrium state to be globally asymptotically stable. This results epidemiologically implies that the COVID-19 will invade the population in respective of the initial conditions (population) considered.

Keywords: Corona virus disease; global; stability.

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1 Introduction

Corona virus popularly known as (COVID-19) is an ongoing pandemic disease caused by severe acute respiratory syndrome Coronavirus 2, [1,2]. The disease was first discovered in Wuhan, China in December 2019 [2] and was declared to be a public health emergency of international concern on the 30th January, 2020 and was identified as a global pandemic by the World Health Organization(WHO) on the 11th March, 2020 [3,4]. 1.94 million Cases were reported as of 14th April, 2020 in over 210 countries of the world, leading to over 121,000 deaths with at least 465,000 recoveries including Nigeria [5,6,7].

Fever, cough, shortness of breath, pneumonia and acute respiratory distress syndrome were identified as possible symptoms of corona virus, [8,9]. The disease has an incubation period of 2-14 days [10]. The virus majorly is contracted during close contact with infected person and also by small droplets produced in the process of sneezing, talking and coughing, [2]. In this present work, we extend the work of Atokolo et al. in [11] by conducting a Global Stability Analysis (GAS) on the Corona virus model.

The concept of global stability is concerned with global properties of a model which can be investigated using Lyapunov function theory. Lyapunov gave a technique that can show if an equilibrium state is stable or unstable through the construction of a Lyapunov function. Lyapunov functions are positive functions that reduce in time along the orbits of a model. The method is advantageous because it sometimes proves stability of a non hyperbolic equilibrium, [12], however, there is no direct method of constructing a Lyapunov functions.

An equilibrium state is asymptotically stable globally if its property holds globally and its domain of attraction is the entire space, [12]. Models such as the ones in [12,13,14], are veritable tools towards studying global stabilities of biological models. where Lyapunov functions were constructed to perform stability analysis on their models.

2 Model Formulation and Procedures

The following assumptions were given by Atokolo et al. in [11]. In modelling the spread of the disease (COVID'19) pandemic.

i. The model incorporates a net inflow of individuals into the susceptible population. This parameter comprises of new births, immigration and emigration.
ii. All classes of the population die naturally.
iii. Disease induced death is considered in the model.
iv. Infected individuals can recover naturally though the rate is assumed to be minimal.
v. The Recovered has permanent immunity for re-infection.
vi. Every individual taken for treatment recovers at a high rate, that is to say the treatment is considered to be effective.
vii. We divide the population into the Susceptible class (S), the Exposed class (E), Quarantine class (Q), Isolated class (J), the Infected class (I), the Infected but treated class (I_T), and the Recovered class(R).

2.1 Mathematical model for the transmission and control of COVID-19

The mathematical model that incorporates the above assumptions as given in [11] is given as:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \wedge - \alpha(1-x)S - \mu S \\
\frac{dE}{dt} &= \alpha(1-x)S - [\theta(1+y) + \beta + \mu]E \\
\frac{dQ}{dt} &= \theta(1+y)E - (\eta + \mu)Q \\
\frac{dJ}{dt} &= \eta Q + \emptyset(1+z)I - (\mu + \sigma + r + \rho)J \\
\frac{dI}{dt} &= \beta E - [\emptyset(1+Z) + \lambda + \sigma + \mu]I \\
\frac{dI_T}{dt} &= \gamma J - (\omega + \mu + \sigma)I_T \\
\frac{dR}{dt} &= \lambda I + \rho J + \omega I_T - \mu R
\end{aligned}\right\} \quad (1)$$

Where $\alpha = \frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_S I_T}{N}$

and $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$

where $x, y, z,$ are control parameters

2.2 Model variables and parameters description

Table 1. Model variables and description

S/N	Variables	Description
1.	S	Susceptible Human
2.	E	Exposed Human `
3.	Q	Quarantined Human
4.	J	Isolated Human
5.	I	Infected Human
6.	I_T	Infected but treated Human
7.	R	Recovered Human

Table 2. Model parameters and description

S/N	Parameters	Description
1	$\wedge$	Recruitment rate
2	α	Force of infection
3	θ	Rate at which the exposed are quarantine
4	η	Rate at which the quarantined are isolated
5	β	Rate at which the exposed are infected
6	$\emptyset$	Rate at which the infected are isolated
7	γ	Treatment rate
8	ρ	Natural recovery rate of the isolated
9	λ	Natural recovery rate of the infected
10	ω	Recovery rate due to treatment
11	σ	Disease induced death rate
12	μ	Natural death rate
13	x	Enlightenment control measures for the susceptible individuals to observe social distance, washing of hands, covering of mouth when talking, coughing and sneezing
14	y	Enlightenment control measure for the exposed to be quarantined
15	Z	Enlightenment control measure for the infected to be isolated

2.3 Endemic disease equilibrium (EE)

At the endemic disease equilibrium, infection exists and as such we let:

$$S = S^*, E = E^*, Q = Q^*, J = J^*, I = I^*, I_T = I_T{}^* \; and \; R = R^*$$

Also at equilibrium,

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dQ}{dt} = \frac{dJ}{dt} = \frac{dI}{dt} = \frac{d\boldsymbol{I_T}}{dt} = \frac{dR}{dt} = 0$$

The EE is represented by $[S^*, E^*, Q^*, J^*, I^*, I_T{}^* \; R^*]$ which is given by:

$$\varepsilon_1 = \left\{ \begin{matrix} \frac{\alpha(1-x)}{[\theta(1+y)+\beta+\mu]E^*}, \frac{\beta}{[\emptyset(1+z)+\lambda+\sigma+\mu]}, \frac{\theta(1+y)E^*}{(\eta+\mu)}, \frac{(\omega+\mu+\sigma)I_T{}^*}{\gamma}, \\ \frac{(\eta+\mu)(\mu+\sigma+\gamma+\rho)(\omega+\mu+\sigma)\, I_T{}^* - \gamma\eta\theta(1+y)E^*}{\emptyset\gamma(\eta+\mu)(1+z)}, \frac{(\lambda+\mu\gamma\lambda)I^*}{[\omega\gamma\mu+\rho\mu\omega(\omega+\mu+\sigma)]}, \frac{\gamma\lambda I^* + [\rho(\omega+\mu+\sigma)+\gamma\omega]I_T{}^*}{\gamma\mu} \end{matrix} \right\} \quad (2)$$

3 Global Stability Analysis of Endemic Equilibrium Point of the Model

Theorem 2: If $R_0 > 1$, the endemic equilibrium point (ε_1) of the model(1) is globally asymptotically stable.

Proof:

To establish the global stability of the endemic equilibrium point ε_1, we construct the following using Lyapunov function.

$$V(S^* E^* Q^* J^* I^* I_T^* R^*) = \left(S - S^* - S^* log\frac{S^*}{S}\right) + \left(E - E^* - E^* log\frac{E^*}{E}\right) + \left(Q - Q^* - Q^* log\frac{Q^*}{Q}\right) + \left(J - J^* - J^* log\frac{J^*}{J}\right) + \left(I - I^* - I^* log\frac{I^*}{I}\right) + \left(I_T - I_T^* - I_T^* log\frac{I_T^*}{I_T}\right) + \left(R - R^* - R^* log\frac{R^*}{R}\right) \quad (3)$$

Where:

$$\left.\begin{aligned}
\frac{dS}{dt} &= \wedge - \left(\frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_5 I_T}{N}\right)(1-x)S - \mu S \\
\frac{dE}{dt} &= \left(\frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_5 I_T}{N}\right)(1-x)S - [\theta(1+y) + \beta + \mu]E \\
\frac{dQ}{dt} &= \theta(1+y)E - (\eta + \mu)Q \\
\frac{dJ}{dt} &= \eta Q + \emptyset(1+Z)I - (\mu + \sigma + r + \rho)J \\
\frac{dI}{dt} &= \beta E - [\emptyset(1+Z) + \lambda + \sigma + \mu]I \\
\frac{dI_T}{dt} &= \gamma J - (\omega + \mu + \sigma)I_T \\
\frac{dR}{dt} &= \lambda I + \rho J + \omega I_T - \mu R
\end{aligned}\right\} \quad (4)$$

Thus:

$$\frac{dV}{dt} = \frac{S - S^*}{S}\left[\wedge - \left(\frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_5 I_T}{N}\right)(1-x)S - \mu S\right] + \frac{E - E^*}{E}\left[\left(\frac{\alpha_1 E + \alpha_2 Q + \alpha_3 J + \alpha_4 I + \alpha_5 I_T}{N}\right)(1-x)S - [\theta(1+y) + \beta + \mu]E\right] + \frac{Q - Q^*}{Q}[\theta(1+y)E - (\eta + \mu)Q] + \frac{J - J^*}{J}[\eta Q + \emptyset(1+Z)I - (\mu + \sigma + r + \rho)J] + \frac{I - I^*}{I}[\beta E - [\emptyset(1+Z) + \lambda + \sigma + \mu]I] + \frac{I_T - I_T^*}{I_T}[\gamma J - (\omega + \mu + \sigma)I_T] + \frac{R - R^*}{R}[\lambda I + \rho J + \omega I_T - \mu R] \quad (5)$$

Therefore:-

$$\frac{dV}{dt} = (S - S^*)\frac{\left[\wedge - \frac{1}{N}(\alpha_1(E - E^*) + \alpha_2(Q - Q^*) + \alpha_3(J - J^*) + \alpha_4(I - I^*) + \alpha_5(I_T - I_T^*))(1-x)(S - S^*) - \mu(S - S^*)\right]}{S} + (E - E^*)\left[\frac{\frac{1}{N}(\alpha_1(E - E^*) + \alpha_2(Q - Q^*) + \alpha_3(J - J^*) + \alpha_4(I - I^*) + \alpha_5(I_T - I_T^*))(1-x)(S - S^*) - [\theta(1+y) + \beta + \mu](E - E^*)}{E}\right] + \frac{(Q - Q^*)[\theta(1+y)(E - E^*) - (\eta + \mu)(Q - Q^*)]}{Q} + \frac{(J - J^*)[\eta(Q - Q^*) + \emptyset(1+Z)(J - J^*) - (\mu + \sigma + r + \rho)(J - J^*)]}{J} + \frac{(I - I^*)[\beta(E - E^*) - [\emptyset(1+Z) + \lambda + \sigma + \mu](I - I^*)]}{I} + \frac{(I_T - I_T^*)[\gamma(J - J^*) - (\omega + \mu + \sigma)(I_T - I_T^*)]}{I_T} + \frac{(R - R^*)[\lambda(I - I^*) + \rho(J - J^*) + \omega(I_T - I_T^*) - \mu(R - R^*)]}{R} \quad (6)$$

Then we have:

$$\frac{dV}{dt}=\frac{1}{S}\Big[\Big[(S-S^*)\wedge-\frac{\alpha_1}{N}(E-E^*)(S-S^*)^2+\frac{\alpha_2}{N}(Q-Q^*)(S-S^*)^2+\frac{\alpha_3}{N}(J-J^*)(S-S^*)^2+\frac{\alpha_4}{N}(I-I^*)(S-S^*)^2+\frac{\alpha_5}{N}(I_T-I_T^*)(S-S^*)^2\Big](1-x)-\mu(S-S^*)^2\Big]+\frac{1}{E}\Big[\frac{\alpha_1}{N}\Big((E-E^*)^2(S-S^*)+\frac{\alpha_2}{N}(E-E^*)(Q-Q^*)(S-S^*)+\frac{\alpha_3}{N}(E-E^*)(J-J^*)(S-S^*)+\frac{\alpha_4}{N}(E-E^*)(I-I^*)(S-S^*)+\frac{\alpha_5}{N}(E-E^*)(I_T-I_T^*)(S-S^*)\Big)(1-x)-[\theta(1+y)+\beta+\mu](E-E^*)\Big]+\frac{1}{Q}[\theta(Q-Q^*)(1+y)(E-E^*)-(\eta+\mu)(Q-Q^*)^2]+\frac{1}{J}[\eta(J-J^*)(Q-Q^*)+\emptyset(1+Z)(I-I^*)(J-J^*)-(\mu+\sigma+r+\rho)(J-J^*)^2]+\frac{1}{I}[\beta(I-I^*)(E-E^*)-[\emptyset(1+Z)+\lambda+\sigma+\mu](I-I^*)^2]+\frac{1}{I_T}[\gamma(I_T-I_T^*)(J-J^*)-(\omega+\mu+\sigma)(I_T-I_T^*)^2]+\frac{1}{R}[\lambda(R-R^*)(I-I^*)+\rho(J-J^*)+\omega(I_T-I_T^*)-\mu(R-R^*)^2] \quad (7)$$

Therefore:

$$\frac{dV}{dt}=\frac{\wedge(1-x)(S-S^*)}{S}-\frac{\alpha_1(1-x)}{N}\frac{(E-E^*)(S-S^*)^2}{S}+\frac{\alpha_2(1-x)}{N}\frac{(Q-Q^*)(S-S^*)^2}{S}+\frac{\alpha_3(1-x)}{N}\frac{(J-J^*)(S-S^*)^2}{S}+\frac{\alpha_4(1-x)}{N}\frac{(I-I^*)(S-S^*)^2}{S}+\frac{\alpha_5(1-x)}{N}\frac{(I_T-I_T^*)(S-S^*)^2}{S}-\frac{\mu(S-S^*)^2}{S}+\frac{\alpha_1(1-x)}{N}\frac{(E-E)^*(S-S^*)}{S}+\frac{\alpha_2(1-x)}{N}\frac{(E-E^*)(Q-Q^*)(S-S^*)}{S}+\frac{\alpha_3(1-x)}{N}\frac{(E-E^*)(J-J^*)(S-S^*)}{S}+\frac{\alpha_4(1-x)}{N}\frac{(E-E^*)(I-I^*)(S-S^*)}{S}+\frac{\alpha_5(1-x)}{N}\frac{(E-E^*)(I_T-I_T^*)(S-S^*)}{S}-\frac{[\theta(1+y)+\beta+\mu](E-E^*)}{E}+\frac{(Q-Q^*)\theta[(1+y)(E-E^*)]}{Q}-\frac{(\eta+\mu)(Q-Q^*)^2}{Q}+\frac{(J-J^*)\eta(Q-Q^*)}{J}+\frac{(J-J^*)\emptyset(1+Z)(I-I^*)}{J}-\frac{(\mu+\sigma+r+\rho)(J-J^*)^2}{J}+\frac{(I-I^*)\beta(E-E^*)}{I}-\frac{(I-I^*)^2[\emptyset(1+Z)+\lambda+\sigma+\mu]}{I}+\frac{(I_T-I_T^*)\gamma(J-J^*)}{I_T}-\frac{(\omega+\mu+\sigma)(I_T-I_T^*)^2}{I_T}+\frac{(R-R^*)\lambda(I-I^*)}{R}+\frac{(R-R^*)\rho(J-J^*)}{R}+\frac{(R-R^*)\omega(I_T-I_T^*)}{R}-\frac{\mu(R-R^*)^2}{R} \quad (8)$$

Collecting the positive and negative terms from equation (8)

We obtain $\frac{dV}{dt}=M_1-M_2$

Where M_1 represents the positive terms and M_2 represents the negative terms in the expression (8)

$$M_1=\frac{(S-S^*)^2}{S}\Big[\frac{\alpha_2(1-x)(Q-Q^*)}{N}+\frac{\alpha_3(1-x)(J-J^*)}{N}+\frac{\alpha_4(1-x)(I-I^*)}{N}+\frac{\alpha_5(1-x)(I_T-I_T^*)}{N}\Big]+\frac{(E-E^*)^2}{E}\frac{\alpha_1}{N}(1-x)(S-S^*)+\frac{(S-S^*)}{S}\wedge(1-x)+\frac{(E-E^*)}{E}\Big[\frac{\alpha_2(1-x)(Q-Q^*)(S-S^*)}{N}+\frac{\alpha_3(1-x)(J-J^*)(S-S^*)}{N}+\frac{\alpha_4(1-x)(I-I^*)(S-S^*)}{N}+\frac{\alpha_5(1-x)(I_T-I_T^*)(S-S^*)}{N}\Big]+\frac{(Q-Q^*)}{Q}\theta(1+y)(E-E^*)+\frac{(J-J^*)}{J}[\eta(Q-Q^*)+\emptyset(1+Z)(I-I^*)]+\frac{(I-I^*)}{I}\beta(E-E^*)+\frac{(I_T-I_T^*)}{I_T}\gamma(J-J^*)+\frac{(R-R^*)}{R}[\gamma(I-I^*)+\rho(J-J^*)+\omega(I_T-I_T^*)] \quad (9)$$

and

$$M_2=\frac{(S-S^*)}{S}\Big[\frac{\alpha_1}{N}(1-x)(E-E^*)+\mu\Big]+\frac{(Q-Q^*)^2}{Q}(\eta+\mu)+\frac{(J-J^*)^2}{J}(\mu+\sigma+\gamma+\rho)+\frac{(I-I^*)^2}{I}(\emptyset(1+Z)+\lambda+\sigma+\mu)+\frac{(I_T-I_T^*)^2}{I_T}(\omega+\mu+\sigma)+\frac{(R-R^*)}{R}\mu+\frac{(E-E^*)}{E}(\theta(1+y)+\beta+\mu) \quad (10)$$

Therefore, if $M_1<M_2$, then $\frac{dV}{dt}$ will be negative definite along the solution path of the system. Thus this implies that $\frac{dV}{dt}<0$, and $\frac{dV}{dt}=0$ only at a point where $S=S^*, E=E^*, Q=Q^*, J=J^*, I=I^*, I_T=I_T^*\ R=R^*$.

Therefore, the largest compact set is $\left\{(S^*,E^*,Q^*,J^*,I^*,I_T^*\ R^*)\in\Omega\frac{dV}{dt}=0\right\}$ is just the singleton set (ε_1) where (ε_1) is the endemic equilibrium point.

According to Lasalle's Invariant Principle in [15], it therefore means that (ε_1) is globally assymptotically stable in Ω if $M_1 < M_2$.

This results epidemiologically implies that the COVID-19 will invade the population in respective of the initial conditions (population) considered.

4 Conclusion

In this paper, we conducted global stability analysis of Corona Virus (COVID'19) mathematical model. Result of the analysis shows that the model endemic equilibrium point is globally asymptotically stable, which epidemiological implies that the disease will be wiped out of the population in respective of the initial condition or the population under consideration.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] "Corona virus disease 2019" World Health Organization.
Retrieved 15th March, 2020.

[2] "Novel Corona virus in China.'' World Health Organization.
Retrieved 19 th April, 2020.

[3] "Statement on the second meeting of the International Health Population (2005) emergency committee regarding the outbreak of Novel Corona virus" World Health Organization; 30th January, 2020.

[4] "Director-general opening remarks at the media briefing on Covid-19. 11th March 2020". World Health Organization.
Retrieved 11th March, 2020.

[5] "Corona virus (COVID-19) global cases by the centre for systems science and engineering at Johns Hopkins University" Johns Hopkins CSSE.
Retrieved 14th April 2020.

[6] "Corona Virus update (live)" World meter ncov2019 live. 10th April, 2020.

[7] "Global COVID-19 case fatality rates" Centre for Evidence-Base Medicine, 17th March 2020.
Retrieved 10th April 2020.

[8] "Symptoms of Novel Corona virus (2019- ncov)" US centres for Disease Control and Prevention; 10th February, 2020.
Retrieved 11th February, 2020.

[9] "Interim Clinical Guidance for Management of Patients with confirmed Corona virus". Centre for Disease Control and Prevention; 4th April, 2020.
Retrieved,11th April, 2020.

[10] "New Corona virus Stables for hours on Surfaces" National Instituted of Health (NIH); 17th March, 2020. Retrieved 24th March, 2020.

[11] Atokolo W, Omale D, Bashir ST, Olayemi KS, Daniel MA, Akpa J. Mathematical model of the transmission dynamics of corona virus disease and its control. Asian Research Journal of Mathematics. 2020;16(11);69-88. ARJOM 63754.

[12] Omale D, Atokolo W, Akpa M. Global stability and sensitivity analysis of transmission dynamics of tuberculosis and its control. Academic Journal of Statistics and Mathematics (AJSM). 2020;6(2). ISSN: 5730-7151.

[13] Atokolo W, Mbah GCE. Modeling the control of zika virus vector population using the Sterile Insect Technology.Hindawi Journal of Applied Mathematics; 2020. Article ID: 6350134.

[14] Omale D, Ojih PB, Atokolo W, Omale AJ, Bolaji B. Mathematical model for the transmission dynamics of HIV and tuberculosis co-infection in Kogi State, Nigeria. Journal of Mathematical and Computational Sciences. 2021;11(5):5580-5613.
ISSN: 1927-5307.

[15] Lasalle J, Lefschetz S. The stability of dynamical systems. SIAM, Philadelphia; 1976.

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