



New Criterion of Robust H_∞ Stabilization for Uncertain Neutral Systems

Jiyong Lu^{1*}

¹College of Science, Tianjin University of Science and Technology, Tianjin 300457, China.

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2021/v17i630304

Editor(s):

(1) Dr. Nikolaos D. Bagis, Aristotle University of Thessaloniki, Greece.

Reviewers:

(1) Neelam Sanjeev Kumar, India.

(2) Fahrettin ÖZBEY, Bitlis Eren University, Turkey.

Complete Peer review History: <https://www.sdiarticle4.com/review-history/71889>

Original Research Article

Received 03 June 2021

Accepted 09 August 2021

Published 16 August 2021

Abstract

The problems of delay-dependent robust stability and stabilization for a class of uncertain neutral systems are investigated in this paper. At first, by constructing a new Lyapunov functional and using the Lyapunov stability theory, a new delay-dependent condition which renders the system with no external disturbance and input to be asymptotically stable is obtained and given by a linear matrix inequality. Then, based on the obtained condition, a state feedback stabilize law is designed, which guarantees closed-loop neutral systems are asymptotically stable for all the permitted uncertainties when the external disturbance is naught, and it can also guarantee the closed-loop systems have H_∞ performance under the external disturbance. The model of neutral systems with both the uncertainty and the disturbance discussed in this paper has rarely been considered before.

Keywords: Neutral system; delay-dependent; robust stability; H_∞ state feedback; linear matrix inequality.

1 Introduction

Time delay phenomenon exists widely in all kinds of systems. At the same time, due to the aging of system components, parameter disturbance and other reasons, all kinds of uncertainties of the system inevitably exist,

*Corresponding author: Email: lujiyong@tust.edu.cn;

which leads to the deterioration of system stability performance. In order to study and solve these problems, robust control theory as the main research method has attracted more and more attention of scholars. Neutral time-delay system is a special case, which contains not only the time-delay term of the system state, but also the time-delay term of the system state derivative. In recent years, the research on the stability and stabilization of neutral systems has aroused the interest of many scholars [1-11]. At present, the stability criteria of neutral systems can be divided into two categories: delay- dependent and delay independent. The delay-dependent stability condition contains the information of the delay constant, while the delay independent stability condition is independent of the delay constant. In reference [1-2], the stability of a class of neutral systems was studied based on the linear inequality technique. The research in [7] shows that when the delay constant is small, the delay independent stability condition usually has strong conservatism, while the delay-dependent stability criterion has weak conservatism. Therefore, the delay-dependent stability criteria for neutral systems are more concerned [2,3,6-8]. In reference [3], the model transformation was introduced to study a class of uncertain neutral systems, and the delay-dependent robust stability conditions are obtained, However, this method needs to define some quadratic cross terms which makes the conditions conservative. In reference [7], the "free weight matrix method" is proposed to study the delay-dependent stability of neutral time-delay systems. The delay-dependent stability conditions with less conservatism are obtained, and the state feedback stabilization law is designed. However, the research in reference [8] shows that the introduction of free weight matrix does not always reduce the conservatism of the obtained conditions. In addition, reference [6-8] did not consider the influence of external interference. The reference [9] investigated the problem of the delay-dependent stability of neutral systems with mixed-delay and time-varying structured uncertainties. It obtains some less conservative criteria by combining the free-weighting matrix technique and Wirtinger-based integral inequality technique. The reference [10] concerned with the exponential stability problem for uncertain neutral systems with mixed time-varying delays and nonlinear perturbations. It considers the upper and lower bounds of interval time-varying delays when constructing Lyapunov functional. The reference [11] discussed the delay-dependent stability for neutral singular systems and developed a novel augmented Lyapunov-Krasovskii functional including less decision variables.

In this paper, by using Lyapunov stability theory and linear matrix inequality technique to construct a new Lyapunov functional, delay-dependent stability conditions for a class of linear uncertain non-forced neutral systems are obtained based on the properties of positive definite matrix. The maximum delay constant to ensure the robust stability of the system can be found by Matlab toolbox. Moreover, a state feedback control law is designed to make the closed-loop system robust and stable. It is further proved that the system satisfies H_∞ norm boundedness when the external disturbance is not zero. Compared with those previous studies, this paper considers not only the stability problem, but also the stabilization problem and the robust H_∞ performance of the system for external disturbances. In this paper, the model of neutral systems with both the uncertainty and the disturbance is considered, which has rarely been discussed before. In particular, a new state feedback stabilize law is designed. The current research is limited to theoretical analysis and has not found a reasonable application, which will be our future work.

2 System Description and Preliminaries

Consider the following linear neutral uncertain time-delay systems:

$$\begin{cases} \dot{x}(t) - J\dot{x}(t-h) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-h) + Bu(t) + B_1w(t), \\ z(t) = Cx(t) + C_1x(t-h), \\ x(t) = 0, \forall t \in [-h, 0], \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the state vector of the system, $u(t) \in R^m$ is the control input vector of the system, $w(t) \in L_2[0, \infty)$ is the external disturbance, $z(t) \in R^p$ is the controlled output vector, $A, A_1, B, B_1, J, C, C_1$ are matrices with proper dimensions, $h > 0$ is a delay constant. $\Delta A, \Delta A_1$ are unknown uncertain terms, and have the corresponding dimension. Assuming that it has the following form:

$$[\Delta A \quad \Delta A_1] = E\Sigma(t)[F_0 \quad F_1]. \quad (2)$$

where E, F_1, F_2 are matrices with proper dimensions, $\Sigma(t)$ is an unknown time-varying function matrix, satisfying the following conditions:

$$\Sigma^T(t)\Sigma(t) \leq I, \quad \forall t \in R \quad (3)$$

When $u(t) = 0, w(t) = 0$, the system (1) without control input and external interference is obtained as follows:

$$\begin{cases} \dot{x}(t) - J\dot{x}(t-h) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-h), \\ z(t) = Cx(t) + C_1x(t-h), \\ x(t) = 0, \quad \forall t \in [-h, 0]. \end{cases} \quad (4)$$

The following lemma will be used in the following discussions and proofs.

Lemma 1 [4]. (Schur complement lemma) Given constant matrices S_1, S_2, S_3 , and $S_1 = S_1^T, 0 < S_2 = S_2^T$, then

$$S_1 + S_3^T S_2^{-1} S_3 < 0 \text{ holds if and only if } \begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0.$$

Lemma 2 [7]. Gives matrices $Q = Q^T, H$ and E with proper dimensions, then $Q + HF(t)E + E^T F^T(t)H^T < 0$ holds for any $F(t)$ satisfying $F^T(t)F(t) \leq I$ if and only if there exists a constant $\varepsilon > 0$ such that $Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$.

3 Robust Stability of Systems

In this section, we will give a sufficient condition for the delay-dependent robust stability of the system (4) by using linear matrix inequalities. For the convenience of discussion, note that $\bar{A} = A + \Delta A, \bar{A}_1 = A_1 + \Delta A_1$. By the system (4), We can get $Y(t) = 0 = J\dot{x}(t-h) - \dot{x}(t) + \bar{A}x(t) + \bar{A}_1x(t-h)$. Thus, $\dot{x}^T(t)MY(t) = 0$ holds for any matrix $M > 0$ with appropriate dimension.

Theorem 1. Assume that $\|J\| \leq 1$, if there is a positive real number $\varepsilon > 0$, symmetric positive definite matrices $P > 0, Q_1 > 0, Q_2 > 0, X_{33} > 0$ and a semi-positive definite matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \geq 0$$

such that the following inequalities hold:

$$\Omega = \begin{bmatrix} \Omega_{11} & A^T P & \Omega_{13} & 0 & PE \\ * & \Omega_{22} & PA_1 & PJ & PE \\ * & * & \Omega_{33} & 0 & JPE \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & -\frac{\varepsilon}{2} I \end{bmatrix} < 0, \quad (5)$$

where

$$\begin{cases} \Omega_{11} = A^T P + PA + Q_1 + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0, \\ \Omega_{22} = -2P + hX_{33} + Q_2, \\ \Omega_{13} = -A^T P J + P A_1 + hX_{12} - X_{13} + X_{23}^T, \\ \Omega_{33} = -(A_1^T P J + J^T P A_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1, \end{cases} \quad (6)$$

then the system (4) is robust stable for all uncertainties satisfying (2) and (3).

Proof: For the system (4), the Lyapunov functional is constructed as follows:

$$V = V_1 + V_2 + V_3 + V_4,$$

where

$$\begin{aligned} V_1 &= (x(t) - Jx(t-h))^T P(x(t) - Jx(t-h)), \\ V_2 &= \int_{t-h}^t x^T(s) Q_1 x(s) ds + \int_{t-h}^t \dot{x}^T(s) Q_2 \dot{x}(s) ds, \\ V_3 &= \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) X_{33} \dot{x}(s) ds d\theta, \\ V_4 &= \int_0^t \int_{\tau-h}^{\tau} \xi^T X \xi ds d\tau, \end{aligned}$$

and

$$\xi = (x^T(\tau), x^T(\tau-h), \dot{x}^T(s))^T, X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix}.$$

Derivation of V along the trajectory of system (4) is as follows:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4.$$

where

$$\begin{aligned} \dot{V}_1 &= (\dot{x}(t) - J\dot{x}(t-h))^T P(x(t) - Jx(t-h)) + (x(t) - Jx(t-h))^T P(\dot{x}(t) - J\dot{x}(t-h)). \\ \text{By } Y(t) = 0 &= J\dot{x}(t-h) - \dot{x}(t) + \bar{A}x(t) + \bar{A}_1 x(t-h), \text{ we can get} \\ \dot{V}_1 &= (\dot{x}(t) - J\dot{x}(t-h))^T P(x(t) - Jx(t-h)) + (x(t) - Jx(t-h))^T P(\dot{x}(t) - J\dot{x}(t-h)) + \\ &\quad \dot{x}^T(t) P Y(t) + Y^T(t) P \dot{x}(t), \\ &= (\bar{A}x(t) + \bar{A}_1 x(t-h))^T P(x(t) - Jx(t-h)) + (x(t) - Jx(t-h))^T P(\bar{A}x(t) + \bar{A}_1 x(t-h)) + \\ &\quad \dot{x}^T(t) P (J\dot{x}(t-h) - \dot{x}(t) + \bar{A}x(t) + \bar{A}_1 x(t-h)) + (J\dot{x}(t-h) - \dot{x}(t) + \bar{A}x(t) + \bar{A}_1 x(t-h))^T P \dot{x}(t) \\ &= x^T(t) (\bar{A}^T P + P \bar{A}) x(t) - x^T(t) \bar{A}^T P J x(t-h) - x^T(t-h) J^T P \bar{A} x(t) + x^T(t-h) \bar{A}_1^T P x(t) + \\ &\quad x^T(t) P \bar{A}_1 x(t-h) - x^T(t-h) (\bar{A}_1^T P J + J^T P \bar{A}_1) x(t-h) + \dot{x}^T(t) P J \dot{x}(t-h) - \dot{x}^T(t) P \dot{x}(t) + \\ &\quad \dot{x}^T(t) P \bar{A} x(t) + \dot{x}^T(t) P \bar{A}_1 x(t-h) + \dot{x}^T(t-h) J^T P \dot{x}(t) - \dot{x}^T(t) P \dot{x}(t) + x^T(t) \bar{A}^T P \dot{x}(t) + \\ &\quad x^T(t-h) \bar{A}_1^T P \dot{x}(t). \\ \dot{V}_2 &= x^T(t) Q_1 x(t) - x^T(t-h) Q_1 x(t-h) + \dot{x}^T(t) Q_2 \dot{x}(t) - \dot{x}^T(t-h) Q_2 \dot{x}(t-h). \end{aligned}$$

$$\begin{aligned} \dot{V}_3 &= h\dot{x}^T(t)X_{33}\dot{x}(t) - \int_{t-h}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds. \\ \dot{V}_4 &= h[x^T(t) \quad x^T(t-h)] \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} + [x^T(t) \quad x^T(t-h)] \begin{bmatrix} X_{13} \\ X_{23} \end{bmatrix} [x(t) - x(t-h)] + \\ & [x^T(t) - x^T(t-h)] \begin{bmatrix} X_{13}^T & X_{23}^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} + \int_{t-h}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds \\ &= x^T(t)(hX_{11} + X_{13} + X_{13}^T)x(t) + x^T(t)(hX_{12} - X_{13} + X_{23}^T)x(t-h) + x^T(t-h)(hX_{12}^T - X_{13}^T + \\ & X_{23})x(t) + x^T(t-h)(hX_{22} - X_{23} - X_{23}^T)x(t-h) + \int_{t-h}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds. \end{aligned}$$

For the convenience of discussion, define $\zeta(t) = (x^T(t), \dot{x}^T(t), x^T(t-h), \dot{x}^T(t-h))^T$, then $\dot{V} = \zeta^T(t)\Xi\zeta(t)$, where

$$\Xi = \begin{bmatrix} \Xi_{11} & \bar{A}^T P & \Xi_{13} & 0 \\ * & -2P + hX_{33} + Q_2 & P\bar{A}_1 & PJ \\ * & * & \Xi_{33} & 0 \\ * & * & * & -Q_2 \end{bmatrix},$$

and

$$\begin{aligned} \Xi_{11} &= \bar{A}^T P + P\bar{A} + Q_1 + hX_{11} + X_{13} + X_{13}^T, \\ \Xi_{13} &= -\bar{A}^T PJ + P\bar{A}_1 + hX_{12} - X_{13} + X_{23}^T, \\ \Xi_{33} &= -(\bar{A}_1^T PJ + J^T P\bar{A}_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T. \end{aligned}$$

According to the Lyapunov stability theory, a sufficient condition for the system (1) with $u(t) = 0, w(t) = 0$, that is the system (4), to be robust asymptotically stable is that there exist real positive definite matrices P, Q_1, Q_2, X_{33} and a semi-positive definite symmetric matrix $X \geq 0$ such that

$$\Xi < 0 \tag{7}$$

Note that there are uncertainties $\Delta A, \Delta B$ in the matrix elements, so we should eliminate the uncertainties to obtain linear matrix inequalities. By substituting uncertainty conditions (2), (3) into (7), we can get

$$\Xi = \Xi_0 + \Pi + \Pi^T < 0, \tag{8}$$

where

$$\Xi_0 = \begin{bmatrix} \Xi_{011} & A^T P & \Xi_{013} & 0 \\ * & -2P + hX_{33} + Q_2 & PA_1 & PJ \\ * & * & \Xi_{033} & 0 \\ * & * & * & -Q_2 \end{bmatrix},$$

With

$$\begin{aligned} \Xi_{011} &= A^T P + PA + Q_1 + hX_{11} + X_{13} + X_{13}^T, \\ \Xi_{013} &= -A^T PJ + PA_1 + hX_{12} - X_{13} + X_{23}^T, \\ \Xi_{033} &= -(A_1^T PJ + J^T PA_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T, \end{aligned}$$

and

$$\Pi = \begin{bmatrix} PE & 0 & PE & 0 \\ PE & 0 & PE & 0 \\ -J^T PE & 0 & -J^T PE & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} Y \begin{bmatrix} F_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

with $Y = \text{diag}\{\Sigma(t), \Sigma(t), \Sigma(t), \Sigma(t)\}$. From Lemma 2, we can see that (8) is equivalent to

$$\Xi_0 + \varepsilon \begin{bmatrix} F_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} F_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & F_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \varepsilon^{-1} O O^T < 0,$$

where

$$O = \begin{bmatrix} PE & 0 & PE & 0 \\ PE & 0 & PE & 0 \\ -J^T PE & 0 & -J^T PE & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

By simplification and arrangement, we can obtain

$$\begin{bmatrix} \tilde{\Xi}_{11} & A^T P + 2\varepsilon^{-1} PEE^T P & \tilde{\Xi}_{13} & 0 \\ * & \tilde{\Xi}_{22} & PA_1 - 2\varepsilon^{-1} PEE^T PJ & PJ \\ * & * & \tilde{\Xi}_{33} & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0,$$

where

$$\begin{aligned} \tilde{\Xi}_{11} &= A^T P + PA + Q_1 + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0 + 2\varepsilon^{-1} PEE^T P, \\ \tilde{\Xi}_{22} &= -2P + hX_{33} + Q_2 + 2\varepsilon^{-1} PEE^T P, \\ \tilde{\Xi}_{13} &= -A^T PJ + PA_1 + hX_{12} - X_{13} + X_{23}^T - 2\varepsilon^{-1} PEE^T PJ, \\ \tilde{\Xi}_{33} &= -(A_1^T PJ + J^T PA_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1 + 2\varepsilon^{-1} J^T PEE^T PJ. \end{aligned}$$

According to Schur complement lemma, the above formula can be changed into:

$$\begin{bmatrix} \Omega_{11} & A^T P & \Omega_{13} & 0 & PE \\ * & \Omega_{22} & PA_1 & PJ & PE \\ * & * & \Omega_{33} & 0 & -J^T PE \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & -\frac{\varepsilon}{2} I \end{bmatrix} < 0. \quad (9)$$

where

$$\begin{aligned}\Omega_{11} &= A^T P + PA + Q_1 + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0, \\ \Omega_{22} &= -2P + hX_{33} + Q_2, \\ \Omega_{13} &= -A^T PJ + PA_1 + hX_{12} - X_{13} + X_{23}^T, \\ \Omega_{33} &= -(A_1^T PJ + J^T PA_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1.\end{aligned}$$

Therefore, the theorem is proved. Actually, the linear matrix inequalities (9) provide a sufficient condition for the system (4) to be robust asymptotically stable.

4 Robust and Stabilization

The system (1) is said to be robustly stabilizable if it has state feedback control $u(t) = Kx(t)$ when $w(t) = 0$ such that the following closed-loop system is stable,

$$\begin{aligned}\dot{x}(t) - J\dot{x}(t-h) &= (A + \Delta A + BK)x(t) + (A_1 + \Delta A_1)x(t-h), \\ x(t) &= 0, \forall t \in [-h, 0].\end{aligned}\tag{10}$$

In this section, we mainly study the robust robustness of the system (1) under the condition $u(t) = Kx(t)$, and give the design method of the state feedback law, that is, find out the state feedback gain matrix to make the closed-loop system (10) asymptotically stable.

Theorem 2. When $w(t) = 0$, the system (1) is stable under the state feedback control $u(t) = Kx(t)$, if there are positive real numbers $\varepsilon > 0, \delta > 0$, symmetric positive definite matrix $P > 0, Q_1 > 0, Q_2 > 0, X_{33} > 0$ and semi

positive definite matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \geq 0$ such that the following inequalities are established,

$$\tilde{\Omega} = \begin{bmatrix} \Omega_{11} & A^T P & \Omega_{13} & 0 & PE & \sqrt{2}PB \\ * & \Omega_{22} & PA_1 & PJ & PE & \frac{1}{\sqrt{2}}PB \\ * & * & \Omega_{33} & 0 & -J^T PE & -\frac{1}{\sqrt{2}}J^T PB \\ * & * & * & -Q_2 & 0 & 0 \\ * & * & * & * & -\frac{\varepsilon}{2}I & 0 \\ * & * & * & * & * & -\delta I \end{bmatrix} < 0,$$

where $\Omega_{11}, \Omega_{22}, \Omega_{13}, \Omega_{33}$ are the same as that in (6), and the state feedback control law is given by $K = \delta^{-1}B^T P$.

Proof: Choose the Lyapunov function

$$V = V_1 + V_2 + V_3 + V_4,$$

where

$$\begin{aligned} V_1 &= (x(t) - Jx(t-h))^T P(x(t) - Jx(t-h)), \\ V_2 &= \int_{t-h}^t x^T(s) Q_1 x(s) ds + \int_{t-h}^t \dot{x}^T(s) Q_2 \dot{x}(s) ds, \\ V_3 &= \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) X_{33} \dot{x}(s) ds d\theta, \\ V_4 &= \int_0^t \int_{\tau-h}^{\tau} \xi^T X \xi ds d\tau. \end{aligned}$$

Let $\tilde{A} = A + \Delta A + BK$, $\tilde{A}_1 = A_1 + \Delta A_1$. By substituting $K = \delta^{-1} B^T P$ into the closed-loop system (10) and applying the conclusion of Theorem 1, we can obtain the following sufficient condition for the system (10) to be robust asymptotically stable:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 \\ * & \Pi_{22} & \Pi_{23} & PJ \\ * & * & \Pi_{33} & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0 \quad (11)$$

where

$$\begin{aligned} \Pi_{11} &= A^T P + PA + Q_1 + 2\delta^{-1} PBB^T P + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0 + 2\varepsilon^{-1} PEE^T P, \\ \Pi_{12} &= A^T P + \delta^{-1} PBB^T P + 2\varepsilon^{-1} PEE^T P, \\ \Pi_{13} &= -A^T PJ + PA_1 - \delta^{-1} PBB^T PJ + hX_{12} - X_{13} + X_{23}^T - 2\varepsilon^{-1} PEE^T PJ, \\ \Pi_{22} &= -2P + hX_{33} + Q_2 + 2\varepsilon^{-1} PEE^T P, \\ \Pi_{23} &= PA_1 - 2\varepsilon^{-1} PEE^T PJ, \\ \Pi_{33} &= -(A_1^T PJ + J^T PA_1) - Q_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1 + 2\varepsilon^{-1} J^T PEE^T PJ. \end{aligned}$$

On the other hand, according to Schur complement lemma, (11) is equivalent to

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 \\ * & \Pi_{22} + \frac{1}{2}\delta^{-1} PBB^T P & \Pi_{23} - \frac{1}{2}\delta^{-1} PBB^T PJ & PJ \\ * & * & \Pi_{33} + \frac{1}{2}\delta^{-1} J^T PBB^T PJ & 0 \\ * & * & * & -Q_2 \end{bmatrix} < 0. \quad (12)$$

Due to $\delta > 0$, it is easy to know that

$$\begin{bmatrix} \frac{1}{2}\delta^{-1} PBB^T P & -\frac{1}{2}\delta^{-1} PBB^T PJ \\ -\frac{1}{2}\delta^{-1} J^T PBB^T P & \frac{1}{2}\delta^{-1} J^T PBB^T PJ \end{bmatrix} \geq 0.$$

Thus, the establishment of (13) can deduce the establishment of (12). Therefore, the closed-loop system (10) is robust and asymptotically stable. That is to say, when $w(t) = 0$, the system (1) is robust and stable under the control $u(t) = \delta^{-1} B^T Px(t)$.

Based on the above theorem, we will show that for any nonzero external disturbance $w(t) \in L_2[0, \infty)$ and a given scalar $\gamma > 0$, $\|z(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$ holds for the system (1) under the control $u(t) = \delta^{-1} B^T P x(t)$.

Theorem 3. For a given constant $\gamma > 0$, there is a state feedback control $u(t) = Kx(t)$ for the system (1) such that when $w(t) = 0$, the closed-loop system is robust and stable, and when $w(t) \neq 0$, $\|z(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$ holds, whose sufficient condition is that there exist positive constants $\varepsilon > 0, \delta > 0$, symmetric positive definite matrices $P > 0, Q_1 > 0, Q_2 > 0, X_{33} > 0$, and a positive semidefinite matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \geq 0,$$

such that

$$\Theta = \begin{bmatrix} \Theta_{11} & A^T P & \Theta_{13} & 0 & PB_1 & PE & \sqrt{2}PB \\ * & \Theta_{22} & PA_1 & PJ & PB_1 & PE & \sqrt{2}PB \\ * & * & \Theta_{33} & 0 & -J^T PB_1 & -J^T PE & -\frac{1}{\sqrt{2}} J^T PB \\ * & * & * & -Q_2 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 & 0 & 0 \\ * & * & * & * & * & -\frac{\varepsilon}{2} I & 0 \\ * & * & * & * & * & * & -\delta I \end{bmatrix}, \quad (13)$$

where

$$\begin{aligned} \Theta_{11} &= A^T P + PA + Q_1 + C^T C + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0, \\ \Theta_{13} &= -A^T PJ + PA_1 + C^T C_1 + hX_{12} - X_{13} + X_{23}^T, \\ \Theta_{22} &= -2P + hX_{33} + Q_2, \\ \Theta_{33} &= -(A_1^T PJ + J^T PA_1) - Q_1 + C_1^T C_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1. \end{aligned}$$

In addition, the state feedback control law is given by $K = \delta^{-1} B^T P$.

Proof: Choose the Lyapunov function

$$V = V_1 + V_2 + V_3 + V_4. \quad (14)$$

where

$$\begin{aligned} V_1 &= (x(t) - Jx(t-h))^T P (x(t) - Jx(t-h)), \\ V_2 &= \int_{t-h}^t x^T(s) Q_1 x(s) ds + \int_{t-h}^t \dot{x}^T(s) Q_2 \dot{x}(s) ds, \\ V_3 &= \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) X_{33} \dot{x}(s) ds d\theta, \\ V_4 &= \int_0^t \int_{\tau-h}^{\tau} \xi^T X \xi ds d\tau. \end{aligned}$$

Due to the establishment of (14), it is easy to deduce the establishment of (11), that is, the closed-loop system is robust and stable under the condition $w(t) = 0$. Then we will prove that when $w(t) \neq 0$, for a given positive constant $\gamma > 0$, $\|z(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$ holds. Select a performance index

$$H = \int_0^{\infty} z^T(t)z(t) - \gamma^2 w^T(t)w(t) dt \quad (15)$$

From the zero initial state, we can get

$$H = \int_0^{\infty} ((z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t))) dt - \lim_{t \rightarrow \infty} V(t).$$

According to the selection of the Lyapunov function, $\lim_{t \rightarrow \infty} V(t) \geq 0$ holds evidently. Thus

$$H \leq \int_0^{\infty} ((z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t))) dt. \quad (16)$$

By substituting $z(t) = Cx(t) + C_1x(t-h)$ into the above formula and deriving (15) along the closed-loop system, we can get the following results:

$$\begin{aligned} z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) &= x^T(t)C^T Cx(t) + x^T(t)C^T C_1x(t-h) + x^T(t-h)C_1^T Cx(t) \\ &+ x^T(t-h)C_1^T C_1x(t-h) + x^T(t)(\tilde{A}^T P + P\tilde{A})x(t) - x^T(t)\tilde{A}^T PJx(t-h) - x^T(t-h)J^T P\tilde{A}x(t) \\ &+ x^T(t-h)\tilde{A}_1^T Px(t) + x^T(t)P\tilde{A}_1x(t-h) - x^T(t-h)(\tilde{A}_1^T PJ + J^T P\tilde{A}_1)x(t-h) + w^T(t)B_1^T Px(t) \\ &- w^T(t)B_1^T PJx(t-h) + x^T(t)PB_1w(t) - x^T(t-h)J^T PB_1w(t) + \dot{x}^T(t)PJ\dot{x}(t-h) - \dot{x}^T(t)P\dot{x}(t) \\ &+ \dot{x}^T(t)P\tilde{A}x(t) + \dot{x}^T(t)P\tilde{A}_1x(t-h) + \dot{x}^T(t)PB_1w(t) + \dot{x}^T(t-h)J^T P\dot{x}(t) - \dot{x}^T(t)P\dot{x}(t) + x^T(t)\tilde{A}^T \\ &P\dot{x}(t) + x^T(t-h)\tilde{A}_1^T P\dot{x}(t) + w^T(t)B_1^T P\dot{x}(t) + x^T(t)Q_1x(t) - x^T(t-h)Q_1x(t-h) + \dot{x}^T(t)Q_2\dot{x}(t) \\ &- \dot{x}^T(t-h)Q_2\dot{x}(t-h) + h\dot{x}^T(t)X_{33}\dot{x}(t) - \int_{t-h}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds + \dot{x}^T(t)PY(t) + Y^T(t)P\dot{x}(t) \\ &+ x^T(t)(hX_{11} + X_{13} + X_{13}^T)x(t) + x^T(t)(hX_{12} - X_{13} + X_{23}^T)x(t-h) + x^T(t-h)(hX_{12}^T - X_{13}^T + X_{23}) \\ &x(t) + x^T(t-h)(hX_{22} - X_{23} - X_{23}^T)x(t-h) + \int_{t-h}^t \dot{x}^T(s)X_{33}\dot{x}(s)ds - \gamma^2 w^T(t)w(t), \end{aligned}$$

where $\tilde{A} = A + \Delta A + BK$, $\tilde{A}_1 = A_1 + \Delta A_1$. By substituting $K = \delta^{-1}B^T P$ into the above formula and according to Lemme 1 and Lemma 2, we can get $z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) = \eta^T(t)\tilde{\Theta}\eta(t)$, where

$$\eta^T(t) = [x^T(t) \quad \dot{x}^T(t) \quad x^T(t-h) \quad \dot{x}^T(t-h) \quad w^T(t)]$$

and

$$\tilde{\Theta} = \begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} & \tilde{\Theta}_{13} & 0 & PB_1 \\ * & -2P + hX_{33} + Q_2 & \tilde{\Theta}_{23} & PJ & PB_1 \\ * & * & \tilde{\Theta}_{33} & 0 & -J^T PB_1 \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & -\gamma^2 \end{bmatrix}$$

with

$$\begin{aligned} \tilde{\Theta}_{11} &= A^T P + PA + Q_1 + C^T C + 2\delta^{-1}PBB^T P + hX_{11} + X_{13} + X_{13}^T + \varepsilon F_0^T F_0 + 2\varepsilon^{-1}PEE^T P, \\ \tilde{\Theta}_{12} &= A^T P + \delta^{-1}PBB^T P + 2\varepsilon^{-1}PEE^T P, \\ \tilde{\Theta}_{13} &= -A^T PJ + PA_1 + C^T C_1 - \delta^{-1}PBB^T PJ + hX_{12} - X_{13} + X_{23}^T - 2\varepsilon^{-1}PEE^T PJ, \\ \tilde{\Theta}_{23} &= PA_1 - 2\varepsilon^{-1}PEE^T PJ \\ \tilde{\Theta}_{33} &= -(A_1^T PJ + J^T PA_1) - Q_1 + C_1^T C_1 + hX_{22} - X_{23} - X_{23}^T + \varepsilon F_1^T F_1 + 2\varepsilon^{-1}J^T PEE^T PJ. \end{aligned}$$

From lemma 2, the establishment of (14) can deduce $\tilde{\Theta} < 0$. Thus, $H < 0$ holds, that is, $\|z(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$. Therefore, the closed-loop system H_∞ has performance index $\gamma > 0$.

Note 1: Theorem 1, 2 and 3 respectively give conditions of delay-dependent robust stability and robust H_∞ stabilization for uncertain time-delay systems. Since they are all given by LMI, they can be used to solve the maximum value of time-delay h (when γ is given) without parameter adjustment. In these theorems, the maximum delay constant which guarantees that the system (1) is robust stable or robust stabilizable can be obtained by solving the following quadratic convex optimization problem:

$$\begin{cases} \text{maximize } h \\ \text{s.t. (8) (or 10) } P > 0, Q_1 > 0, Q_2 > 0, X_{33} > 0, \\ \varepsilon > 0 \text{ and } X \geq 0 \text{ (or } \delta > 0). \end{cases}$$

This kind of optimization problem can be solved by Matlab toolbox.

Note 2: In fact, we can also solve the smallest γ satisfying the condition in Theorem 3 by giving a delay constant. The method is also to use the toolbox in MATLAB. This kind of problem is also called the optimal H_∞ design problem. Since the methods are similar, all discussions are omitted.

5 Conclusion

In this paper, the problems of robust stability and stabilization for linear uncertain neutral time-delay systems are studied. By constructing a new Lyapunov function and using the technique of linear matrix inequality, a delay-dependent robust stability criterion for a class of uncertain neutral systems with external disturbance and zero control input is given. And then using the results obtained, the problem of robust stabilization for a class of linear uncertain neutral systems without disturbance is discussed. The calculation method of feedback gain matrix is given, and a sufficient condition for delay-dependent robust stability of the corresponding closed-loop system is obtained. Furthermore, the problem of H_∞ robust stabilization for a class of linear uncertain neutral systems with disturbance is studied, and a sufficient condition of robust stabilization for systems with H_∞ performance index is obtained.

Our future work is to construct more Lyapunov functionals in order to obtain more stability conditions. In this paper, the research on neutral systems with the uncertainty and the interference is only theoretical analysis. Its specific application in practice will be another topic to be studied in our future work.

Competing Interests

Author has declared that no competing interests exist.

References

- [1] Chen JD, Lien CH, Fan KK, Chou JH. Criteria for asymptotic stability of a class of neutral systems via a LMI approach [J]. IEE Proc. control theory Appl. 2001;148(6):442-447.
- [2] Fan KK, Chen JD, Lien CH, Hsieh JG. Delay-dependent stability criterion for neutral time-delay systems via linear matrix inequality approach [J]. Journal of Mathematic Analysis and Applications. 2002;273(2): 580-589.
- [3] Park Ju-Hyun. A new delay-dependent criterion for neutral systems with multiple delays [J]. Journal of Computational and Applied Mathematics. 2001;136(1-2):177-184.

- [4] Park J. H. On the design of observer-based controller of linear neutral delay-differential systems [J]. Applied Mathematics and Computation. 2004;150(1):195-202.
- [5] Xu S, Lam J, Yang C. H_∞ and positive-real control for linear neutral delay systems [J]. IEEE Trans. Automatic Control. 2001;46(8):1321-1326.
- [6] Han Qinglong. Robust stability of uncertain delay-differential systems of neutral type [J]. Automatica. 2002;38(4):719-723.
- [7] Wu M, He Y, She J. New delay-dependent stability criteria and stabilizing method for neutral systems [J]. IEEE Trans. Auto. Contr. 2004;49(12):2266-2271.
- [8] Wu M, Zhang XM. New criteria on delay-dependent robust stability for uncertain neutral systems [C]. Journal of Systems Engineering and Electronics. 2005;16(2):351-355.
- [9] Ding L, He Y, Wu M, Zhang Z. New results of mixed-delay-dependent robust stability criteria for uncertain neutral systems [C]. Pro. of the 35th Chinese Control Conference. 2016;1415-1419.
- [10] Wu Q. Improved exponential stability criteria for neutral system with mixed time-varying delays and nonlinear perturbations [J]. Advances in Applied Mathematics. 2021;10(1):238-247.
- [11] Chen W, Gao F, She J, Xia W. Further results on delay-dependent stability for neutral singular systems via state decomposition method [J]. Chaos Solitons & Fractals. 2020;141:110408.

© 2021 Lu; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<https://www.sdiarticle4.com/review-history/71889>