# Assessment of Required Sample Sizes for Estimating Proportions 

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Authors' contributions
This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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## Method Article

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#### Abstract

When estimating a population proportion $p$ within margin of error $m$, a preliminary sample of size $n$ is taken to produce a preliminary sample proportion $y / n$, which is then used to determine the required sample size $(y / n)(1-y / n)(z / m)^{2}$, where $z$ is the critical value for a given level of confidence. The population is assumed to be infinite, so these $\operatorname{Bernoulli}(p)$ observations are mutually independent. Upon taking a new sample based on the required sample size, the coverage probabilities on $p$ are determined exactly for various values of $m, n, p$, and $z$, using a commonly-used formula for a confidence interval on $p$. The coverage probabilities tend


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to be somewhat smaller than their nominal values, and tend to be a lot smaller when $n p$ or $n(1-p)$ is small, which would result in anti-conservative confidence intervals. As a more minor conclusion, since the given margin of error $m$ is not relative to the population proportion $p$, then the required sample size is larger for values of $p$ nearest to 0.5 . The mean and standard deviation of the required sample size are also computed exactly to provide prospective, regarding just how large or how small these required sample sizes need to be.

Keywords: Bernoulli distribution; binomial distribution; sample size determination; confidence interval.

## 2020 Mathematics Subject Classification: 62F10.

## 1 Introduction

Determining the appropriate sample size is an important component of research design, as it directly influences the credibility and utility of study outcomes; it impacts both a study's ability to find meaningful effects and the accuracy of the estimates derived from the data. If the sample size is insufficient, valid conclusions about the data often cannot be made. This introduction synthesizes insights from a range of sources, focusing on the methodologies for determining sample sizes for means and proportions. These approaches predicate the need for the evaluation of the common formula for sample size determination using the coverage probabilities we are evaluating. Accurately determining sample size is crucial in probability as it significantly impacts the statistical power of a study. Foundational principles guide researchers in the statistical community, underscoring the theoretical underpinnings necessary for understanding sample size calculations and their implications for research outcomes using a commonly-used equation [1, 2, 3]. The importance of sample size determination is studied across various study designs and its impact on the validity of research findings [4]. This work provides a practical perspective on the challenges and considerations involved in sample size estimation, offering valuable guidance for researchers.

The complexities associated with calculating sample size for two proportions is addressed, highlighting how the choice of formula and software can significantly influence the calculated sample sizes [5]. This shows the variability and potential inconsistencies in sample size determination practices.

Sample size estimation for health and social science research is explored, presenting principles and considerations tailored to different study designs [6]. This review contributes to a deeper understanding of the factors influencing sample size decisions and their implications for research in these fields.

Methodological insights into sample size calculation for comparing proportions and estimating intraclass correlation coefficients, respectively, are provided [7, 8]. These contributions highlight the mathematical and statistical considerations essential for accurate sample size determination in specific statistical analyses. An approach to determine the optimal sample size for clinical trials, accounting for the population size, is proposed [9]. This approach emphasizes the significance of incorporating broader population characteristics into sample size calculations, providing a nuanced perspective that improves upon conventional methodologies.

The foundational aspects of calculating sample sizes in clinical research is discussed [10]. This article articulates the basic principles underlying sample size calculations, such as the importance of specifying the margin of error, confidence level, and the expected effect size. A method for estimating population proportions is presented, highlighting the potential for significant improvements in efficiency compared to traditional estimators [11]. This article shows the importance of statistical techniques in sample size calculation. Having a sufficient sample size is needed when showing the prevalence of smoking, heart disease, diabetes, and other matters related to health [12]. Showing differences between users and nonusers of electronic cigarettes regarding heart rate, blood pressure, and oral temperature also requires a sufficient sample size [13].

Our paper seeks to bridge the gap between the theoretical and practical aspects of sample size determination by evaluating a commonly-used formula for sample size through the generation of coverage probabilities. This approach contrasts with the existing literature which predominantly focuses on deriving sample sizes for specific situations or using software tools for estimation. We are examining the performance of this particular formula under different scenarios. By considering values producing extra low coverage probabilities, such as $94.0 \%$ or lower when the nominal level is $95 \%$, we seek to uncover potential limitations and biases associated with using this formula in practical settings and applied research practices.

## 2 Method for Determining Coverage Probabilities

A common question in research is what sample size, $N$, is required when estimating a population proportion or Bernoulli probability, $p$, for a given value of $m$, the margin of error, and nominal level of confidence, often set to $95 \%$. The required conservative sample size is

$$
\begin{equation*}
N=0.25(z / m)^{2}, \tag{2.1}
\end{equation*}
$$

where $z$ is the standard normal critical value and is 1.96 for $95 \%$ confidence. Note that a confidence interval on $p$ is often defined to be

$$
\begin{equation*}
\widehat{p} \pm z \sqrt{\widehat{p}(1-\widehat{p}) / N}, \tag{2.2}
\end{equation*}
$$

where $\widehat{p}$ is the sample proportion of successes based on $N$ independent Bernoulli observations.
If a small preliminary sample of size $n$ produces $y$ Bernoulli success and ( $n-y$ ) Bernoulli failures, then a preliminary estimate of $p$ is $p^{*}=y / n$. When $p$ is near 0 or 1 , then an approach more efficient than using Equation 2.1 is setting the required sample size to $N_{y}=\left\lceil p^{*}\left(1-p^{*}\right)(z / m)^{2}\right\rceil$, which depends on $y$, where $\lceil\eta\rceil$ is the ceiling function which rounds any number $\eta$ upward to its nearest integer. In the extremely rare situation where $y=0$ or $y=n$, we redefine the required sample size of the new sample to be $N_{y}=1$ rather than $N_{y}=0$. Therefore,

$$
\begin{equation*}
N_{y}=\max \left\{\left\lceil\frac{y}{n}\left(1-\frac{y}{n}\right)\left(\frac{z}{m}\right)^{2}\right\rceil, 1\right\} . \tag{2.3}
\end{equation*}
$$

Once the required sample size, $N_{y}$, is determined for the new sample, then the final estimate of $p$ is simply $\widehat{p}=x / N_{y}$, where $x$ is the number of Bernoulli successes from the new sample of size $N_{y}$. The required sample of $N_{y}$ Bernoulli observations for the new sample does not include the $n$ Bernoulli observations from the preliminary sample, and all Bernoulli observations are assumed to be independently sampled with common mean, $p$.

Conditional on $y$, the conditional coverage probability using a new sample is the weighted average of $1\left(\mid x / N_{y}-\right.$ $p \mid<m$ ), where the weights are the Binomial probabilities evaluated at $x$ for a sample of size $N_{y}$ and the given probability $p$. The indicator function $1(A)$ is defined to be one if the event $A$ is true and zero otherwise. Therefore, conditional on $y$, the conditional coverage probability is

$$
\begin{equation*}
\sum_{x=0}^{N_{y}} 1\left(\left|\frac{x}{N_{y}}-p\right|<m\right)\binom{N_{y}}{x} p^{x}(1-p)^{N_{y}-x} \tag{2.4}
\end{equation*}
$$

The values of $y$, the number of successes in the preliminary sample, are weighted according to the Binomial probabilities evaluated at $y$ for a preliminary sample of size $n$ and the given probability $p$. Thus, for given values
of $m, p, z$, and preliminary sample size $n$, the unconditional coverage probability is

$$
\begin{equation*}
\sum_{y=0}^{n}\left[\sum_{x=0}^{N_{y}} 1\left(\left|\frac{x}{N_{y}}-p\right|<m\right)\binom{N_{y}}{x} p^{x}(1-p)^{N_{y}-x}\right]\binom{n}{y} p^{y}(1-p)^{n-y} \tag{2.5}
\end{equation*}
$$

where the values of $N_{y}$ are defined by Equation 2.3.
Thus, the unconditional mean of the required sample size $N_{y}$ is

$$
\begin{align*}
E\left(N_{y}\right) & =\sum_{y=0}^{n} N_{y}\binom{n}{y} p^{y}(1-p)^{n-y} \\
& =\sum_{y=0}^{n} \max \left\{\left\lceil\frac{y}{n}\left(1-\frac{y}{n}\right)\left(\frac{z}{m}\right)^{2}\right\rceil, 1\right\}\binom{n}{y} p^{y}(1-p)^{n-y} \tag{2.6}
\end{align*}
$$

and the unconditional second population moment of $N_{y}$ is

$$
\begin{align*}
E\left(N_{y}^{2}\right) & =\sum_{y=0}^{n} N_{y}^{2}\binom{n}{y} p^{y}(1-p)^{n-y} \\
& =\sum_{y=0}^{n} \max \left\{\left\lceil\frac{y}{n}\left(1-\frac{y}{n}\right)\left(\frac{z}{m}\right)^{2}\right\rceil^{2}, 1\right\}\binom{n}{y} p^{y}(1-p)^{n-y}, \tag{2.7}
\end{align*}
$$

based on Equation 2.3.
Therefore, the unconditional standard deviation of $N_{y}$ is

$$
\begin{equation*}
\sigma_{N_{y}}=\sqrt{E\left(N_{y}^{2}\right)-\left[E\left(N_{y}\right)\right]^{2}} . \tag{2.8}
\end{equation*}
$$

The $R$-code used to produce the coverage probabilities as defined by Equation 2.5, along with the unconditional mean and standard deviation of the required sample size $N_{y}$ as defined by Equations 2.6 and 2.8, is shown below. This $R$-code, therefore, produced all of the results in the tables below, and the required sample size is abbreviated as $N$. Hence, these results are based on exact calculation, not simulation.

```
coverage <- function( n=100, p=0.5, m=0.01, nom.prob=0.95 ) {
# INPUT
# 'n' is the preliminary sample size.
# 'p' is the true probability of success.
# 'm' is the desired margin of error.
# 'nom.prob' is the nominal probability.
# OUTPUT
# 'coverage.prob' is the true coverage probability.
# 'mean.N' is the average required sample size.
# 'sd.N' is the standard deviation of the required sample size.
z <- qnorm( (nom.prob+1)/2 )
```

```
coverage.prob <- O ; mean.N <- O ; mean.N.squared <- 0
for (y in 0:n) {
    N <- max( ceiling( y*(n-y)*(z/n/m)^2 ), 1 ) ; x <- 0:N
    coverage.prob <- coverage.prob + sum( ( abs(x/N-p) <= m ) *
        dbinom( x, N, p) ) * dbinom(y,n,p)
    mean.N <- mean.N + N * dbinom(y,n,p)
    mean.N.squared <- mean.N.squared + N^2 * dbinom(y,n,p) }
sd.N <- sqrt( mean.N.squared - mean.N^2)
return( list( coverage.prob=coverage.prob, mean.N=mean.N, sd.N=sd.N ) ) }
```

Values selected for margin of error are $m=0.01, m=0.02$, and $m=0.03$, corresponding to Tables 1,2 , and 3 , respectively. Nominal probabilities are set to $90 \%, 95 \%$, and $99 \%$. Preliminary sample sizes are $n=25,50,75$, and 100. The population proportion was set to $p=0.05,0.1,0.2,0.3,0.4$, and 0.5 , noting that selecting values of $p$ above 0.5 would be redundant due to symmetry.

Table 1. Coverage probabilities for margin of error of $1 \%$

| $m=0.01$ | Nominal Probab. is 0.9 |  | Nominal |  | Probab. is 0.95 | Nominal Probab. is 0.99 |  |  |  |  |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Coverage | Mean | SD |  |  |  |  |  |  |
| $p$ | $n$ | Probab. | of $N_{y}$ | of $N_{y}$ | Probab. | Mean | SD |  |  |  |
| of $N_{y}$ | of $N_{y}$ | Coverage | Probab. | Mean <br> of $N_{y}$ | SD <br> of $N_{y}$ |  |  |  |  |  |
| 0.05 | 25 | 0.6629 | 1234 | 1021 | 0.6884 | 1753 | 1450 | 0.7148 | 3026 | 2505 |
| 0.05 | 50 | 0.7991 | 1260 | 736 | 0.8494 | 1789 | 1046 | 0.8965 | 3089 | 1806 |
| 0.05 | 75 | 0.8472 | 1268 | 605 | 0.8973 | 1801 | 859 | 0.9498 | 3110 | 1484 |
| 0.05 | 100 | 0.8662 | 1273 | 526 | 0.9157 | 1807 | 746 | 0.9653 | 3121 | 1289 |
| 0.1 | 25 | 0.8114 | 2338 | 1254 | 0.8593 | 3320 | 1780 | 0.9032 | 5733 | 3075 |
| 0.1 | 50 | 0.8670 | 2387 | 903 | 0.9201 | 3389 | 1281 | 0.9687 | 5853 | 2213 |
| 0.1 | 75 | 0.8805 | 2403 | 741 | 0.9332 | 3412 | 1052 | 0.9792 | 5892 | 1818 |
| 0.1 | 100 | 0.8847 | 2411 | 644 | 0.9372 | 3423 | 914 | 0.9828 | 5912 | 1579 |
| 0.2 | 25 | 0.8729 | 4156 | 1269 | 0.9254 | 5901 | 1803 | 0.9736 | 10192 | 3113 |
| 0.2 | 50 | 0.8884 | 4243 | 908 | 0.9403 | 6024 | 1289 | 0.9847 | 10404 | 2227 |
| 0.2 | 75 | 0.8925 | 4272 | 744 | 0.9440 | 6065 | 1057 | 0.9868 | 10475 | 1825 |
| 0.2 | 100 | 0.8944 | 4286 | 646 | 0.9457 | 6085 | 917 | 0.9878 | 10510 | 1584 |
| 0.3 | 25 | 0.8865 | 5455 | 1003 | 0.9386 | 7745 | 1424 | 0.9842 | 13376 | 2459 |
| 0.3 | 50 | 0.8943 | 5569 | 706 | 0.9450 | 7906 | 1002 | 0.9877 | 13655 | 1730 |
| 0.3 | 75 | 0.8962 | 5606 | 575 | 0.9469 | 7960 | 816 | 0.9886 | 13748 | 1410 |
| 0.3 | 100 | 0.8977 | 5625 | 498 | 0.9479 | 7987 | 706 | 0.9890 | 13794 | 1220 |
| 0.4 | 25 | 0.8907 | 6234 | 623 | 0.9437 | 8851 | 885 | 0.9873 | 15287 | 1529 |
| 0.4 | 50 | 0.8954 | 6364 | 410 | 0.9471 | 9036 | 582 | 0.9889 | 15606 | 1005 |
| 0.4 | 75 | 0.8973 | 6407 | 326 | 0.9482 | 9097 | 462 | 0.9893 | 15712 | 798 |
| 0.4 | 100 | 0.8979 | 6429 | 278 | 0.9485 | 9128 | 395 | 0.9895 | 15765 | 682 |
| 0.5 | 25 | 0.8923 | 6494 | 375 | 0.9449 | 9220 | 532 | 0.9881 | 15924 | 919 |
| 0.5 | 50 | 0.8963 | 6629 | 189 | 0.9476 | 9412 | 269 | 0.9892 | 16256 | 465 |
| 0.5 | 75 | 0.8982 | 6674 | 127 | 0.9488 | 9476 | 180 | 0.9895 | 16367 | 311 |
| 0.5 | 100 | 0.8981 | 6697 | 95 | 0.9487 | 9508 | 135 | 0.9896 | 16422 | 233 |

Coverage probabilities smaller than $86 \%, 92 \%$, and $97 \%$ regarding nominal probabilities of $90 \%, 95 \%$, and $99 \%$, respectively, are in yellow.
Coverage probabilities within $0.5 \%$ of the nominal probabilities are in pink.

## 3 Results and Discussion

Every coverage probability in Tables 1,2 , and 3 fails to achieve the nominal probability of $90 \%, 95 \%$, or $99 \%$, although in many cases the difference between the coverage and nominal probabilities is evident only in the third significant digit. When $\min \{n p, n(1-p)\}$, the mean number preliminary Bernoulli success or failures, is small, the coverage probability can be substantially lower than the nominal probability, as shown in yellow in the tables. This difference in probabilities is only slightly affected by the margin of error being $m=0.01,0.02$, or 0.03 . Therefore, Tables 1,2 , and 3 produce fairly similar results. For large values of $\min \{n p, n(1-p)\}$, the coverage probability tends to be extremely close to the nominal probability, as shown in pink.

Table 2. Coverage probabilities for margin of error of $\mathbf{2 \%}$

| $m=0.02$ |  | Nominal Probab. is 0.9 |  |  | Nominal Probab. is 0.95 |  |  | Nominal Probab. is 0.99 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | $n$ | Coverage Probab. | Mean of $N_{y}$ | $\begin{gathered} \text { SD } \\ \text { of } N_{y} \end{gathered}$ | Coverage Probab. | Mean of $N_{y}$ | $\begin{gathered} \text { SD } \\ \text { of } N_{y} \end{gathered}$ | Coverage Probab. | Mean of $N_{y}$ | $\begin{gathered} \mathrm{SD} \\ \text { of } N_{y} \end{gathered}$ |
| 0.05 | 25 | 0.6702 | 309 | 255 | 0.6850 | 438 | 362 | 0.7141 | 757 | 626 |
| 0.05 | 50 | 0.8229 | 315 | 184 | 0.8518 | 448 | 261 | 0.8961 | 773 | 451 |
| 0.05 | 75 | 0.8550 | 317 | 151 | 0.8950 | 450 | 215 | 0.9506 | 778 | 371 |
| 0.05 | 100 | 0.8716 | 318 | 131 | 0.9154 | 452 | 187 | 0.9665 | 780 | 322 |
| 0.1 | 25 | 0.8153 | 585 | 314 | 0.8574 | 830 | 445 | 0.9045 | 1433 | 769 |
| 0.1 | 50 | 0.8693 | 597 | 226 | 0.9184 | 848 | 320 | 0.9697 | 1463 | 553 |
| 0.1 | 75 | 0.8828 | 601 | 185 | 0.9320 | 853 | 263 | 0.9790 | 1473 | 454 |
| 0.1 | 100 | 0.8842 | 603 | 161 | 0.9369 | 856 | 229 | 0.9830 | 1478 | 395 |
| 0.2 | 25 | 0.8769 | 1040 | 317 | 0.9268 | 1476 | 451 | 0.9739 | 2548 | 778 |
| 0.2 | 50 | 0.8912 | 1061 | 227 | 0.9404 | 1506 | 322 | 0.9847 | 2601 | 557 |
| 0.2 | 75 | 0.8936 | 1068 | 186 | 0.9439 | 1517 | 264 | 0.9867 | 2619 | 456 |
| 0.2 | 100 | 0.8968 | 1072 | 161 | 0.9456 | 1522 | 229 | 0.9878 | 2628 | 396 |
| 0.3 | 25 | 0.8865 | 1364 | 251 | 0.9384 | 1937 | 356 | 0.9843 | 3344 | 615 |
| 0.3 | 50 | 0.8933 | 1392 | 176 | 0.9448 | 1977 | 250 | 0.9878 | 3414 | 433 |
| 0.3 | 75 | 0.8961 | 1402 | 144 | 0.9468 | 1990 | 204 | 0.9887 | 3437 | 353 |
| 0.3 | 100 | 0.8967 | 1407 | 124 | 0.9474 | 1997 | 177 | 0.9890 | 3449 | 305 |
| 0.4 | 25 | 0.8916 | 1559 | 156 | 0.9438 | 2213 | 221 | 0.9874 | 3822 | 382 |
| 0.4 | 50 | 0.8968 | 1591 | 103 | 0.9474 | 2259 | 146 | 0.9890 | 3902 | 251 |
| 0.4 | 75 | 0.8983 | 1602 | 81 | 0.9483 | 2275 | 116 | 0.9894 | 3928 | 200 |
| 0.4 | 100 | 0.8985 | 1608 | 70 | 0.9488 | 2282 | 99 | 0.9895 | 3942 | 170 |
| 0.5 | 25 | 0.8921 | 1624 | 94 | 0.9432 | 2305 | 133 | 0.9882 | 3982 | 230 |
| 0.5 | 50 | 0.8965 | 1658 | 47 | 0.9463 | 2353 | 67 | 0.9892 | 4064 | 116 |
| 0.5 | 75 | 0.8984 | 1669 | 32 | 0.9478 | 2369 | 45 | 0.9895 | 4092 | 78 |
| 0.5 | 100 | 0.8993 | 1675 | 24 | 0.9482 | 2377 | 34 | 0.9896 | 4106 | 58 |

Coverage probabilities smaller than $86 \%, 92 \%$, and $97 \%$ regarding nominal probabilities of $90 \%, 95 \%$, and $99 \%$, respectively, are in yellow.

Coverage probabilities within $0.5 \%$ of the nominal probabilities are in pink.

The mean of the $N_{y}$, the required sample size, is larger for the values of $p$ closest to 0.5 , as anticipated, and the preliminary sample sizes $n$ have little impact on the mean of $N_{y}$. However, the standard deviation of $N_{y}$ decreases for the larger values of $n$, as anticipated. The larger values of the margin of error obviously produce smaller values of the mean of $N_{y}$.

Table 3. Coverage probabilities for margin of error of $\mathbf{3 \%}$

| $m=0.03$ |  | Nominal Probab. is 0.9 |  |  | Nominal Probab. is 0.95 |  |  | Nominal Probab. is 0.99 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p$ | $n$ | Coverage Probab. | $\begin{aligned} & \text { Mean } \\ & \text { of } N_{y} \end{aligned}$ | $\begin{gathered} \text { SD } \\ \text { of } N_{y} \end{gathered}$ | Coverage Probab. | $\begin{aligned} & \text { Mean } \\ & \text { of } N_{y} \end{aligned}$ | $\begin{gathered} \mathrm{SD} \\ \text { of } N_{y} \end{gathered}$ | Coverage Probab. | Mean of $N_{y}$ | $\begin{gathered} \text { SD } \\ \text { of } N_{y} \end{gathered}$ |
| 0.05 | 25 | 0.6628 | 138 | 113 | 0.6936 | 195 | 161 | 0.7147 | 337 | 278 |
| 0.05 | 50 | 0.7890 | 140 | 82 | 0.8562 | 199 | 116 | 0.8996 | 344 | 201 |
| 0.05 | 75 | 0.8665 | 142 | 67 | 0.8892 | 200 | 96 | 0.9513 | 346 | 165 |
| 0.05 | 100 | 0.8740 | 142 | 58 | 0.9269 | 201 | 83 | 0.9669 | 347 | 143 |
| 0.1 | 25 | 0.8072 | 260 | 139 | 0.8552 | 369 | 198 | 0.9026 | 637 | 341 |
| 0.1 | 50 | 0.8721 | 266 | 100 | 0.9216 | 377 | 142 | 0.9674 | 651 | 246 |
| 0.1 | 75 | 0.8814 | 268 | 82 | 0.9320 | 380 | 117 | 0.9784 | 655 | 202 |
| 0.1 | 100 | 0.8910 | 268 | 72 | 0.9388 | 381 | 102 | 0.9824 | 657 | 175 |
| 0.2 | 25 | 0.8770 | 462 | 141 | 0.9272 | 656 | 200 | 0.9734 | 1133 | 346 |
| 0.2 | 50 | 0.8916 | 472 | 101 | 0.9406 | 670 | 143 | 0.9848 | 1157 | 248 |
| 0.2 | 75 | 0.8958 | 475 | 83 | 0.9446 | 674 | 117 | 0.9869 | 1164 | 203 |
| 0.2 | 100 | 0.8964 | 477 | 72 | 0.9460 | 676 | 102 | 0.9880 | 1168 | 176 |
| 0.3 | 25 | 0.8883 | 607 | 111 | 0.9400 | 861 | 158 | 0.9844 | 1487 | 273 |
| 0.3 | 50 | 0.8951 | 619 | 78 | 0.9462 | 879 | 111 | 0.9878 | 1518 | 192 |
| 0.3 | 75 | 0.8972 | 623 | 64 | 0.9473 | 885 | 91 | 0.9885 | 1528 | 157 |
| 0.3 | 100 | 0.8978 | 626 | 55 | 0.9483 | 888 | 78 | 0.9889 | 1533 | 135 |
| 0.4 | 25 | 0.8895 | 693 | 69 | 0.9431 | 984 | 98 | 0.9875 | 1699 | 170 |
| 0.4 | 50 | 0.8947 | 708 | 45 | 0.9469 | 1004 | 65 | 0.9889 | 1734 | 112 |
| 0.4 | 75 | 0.8969 | 712 | 36 | 0.9482 | 1011 | 51 | 0.9894 | 1746 | 89 |
| 0.4 | 100 | 0.8970 | 715 | 31 | 0.9482 | 1015 | 44 | 0.9896 | 1752 | 76 |
| 0.5 | 25 | 0.8938 | 722 | 42 | 0.9439 | 1025 | 59 | 0.9883 | 1770 | 102 |
| 0.5 | 50 | 0.8958 | 737 | 21 | 0.9479 | 1046 | 30 | 0.9891 | 1807 | 52 |
| 0.5 | 75 | 0.8971 | 742 | 14 | 0.9477 | 1053 | 20 | 0.9895 | 1819 | 34 |
| 0.5 | 100 | 0.8964 | 745 | 11 | 0.9485 | 1057 | 15 | 0.9895 | 1825 | 26 |

Coverage probabilities smaller than $86 \%, 92 \%$, and $97 \%$ regarding nominal probabilities of $90 \%, 95 \%$, and $99 \%$, respectively, are in yellow.

Coverage probabilities within $0.5 \%$ of the nominal probabilities are in pink.

## 4 Conclusions

A preliminary sample of independent Bernoulli random variables may be taken to determine the required sample size for estimating the Bernoulli mean, $p$, within a given margin of error for a fixed level of confidence. When the mean number of successes or failures of this Bernoulli random variable in the preliminary sample is no more than 5 , then the coverage probabilities tend to be severely smaller than their nominal levels, leading to severely anti-conservative confidence intervals. When the mean number of success or failures is between 5 and 15, the coverage probabilities tend to be only somewhat smaller. When the mean number of success or failures is larger than 15 , the coverage probabilities tend to be extremely close and only slightly smaller than their nominal levels. Therefore, in the preliminary sample, taking a preliminary sample size to ensure that $n p$ and $n(1-p)$ both exceed 15 is crucial when trusting the commonly-used equation 2.3 for determining the required sample size. These conclusions herein are limited to situations where equation 2.3 is used to determine the required sample size. For example, future research might involve replicating these studies for finite population sizes, in which case equation 2.3 would be replaced by an equation which includes a finite population correction [14]. The mean and standard deviation of the required sample sizes $N_{y}$ are listed in the table mainly for perspective of the
reader, but do show that they are only slightly impacted by the preliminary sample size $n$ for a given margin of error $m$ and nominal probability.

## Disclaimer (Artificial Intelligence)

Authors hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscript.

## Competing Interests

The authors have declared that no competing interests exist.

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