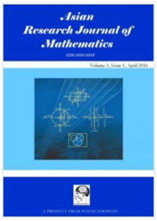

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On $\tau_1\tau_2$-$\bar{g}$-Closed Sets in Bitopological Spaces

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Authors' contributions

This work was carried out in collaboration between both authors. Author PE designed the study and wrote the protocol. Author KV wrote the first draft of the manuscript and managed the analysis of the study. Both authors read and approved the final manuscript.

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Abstract

In this paper, we introduced and studied a new kind of generalized closed set called $\tau_1\tau_2$-$\bar{g}$-closed set in a bitopological space (X, τ_1, τ_2). The properties of this $\tau_1\tau_2$-$\bar{g}$-closed set are studied and compared with some of the corresponding generalized closed sets in general topological spaces and bitopological spaces. We also defined the $\tau_1\tau_2$-$\bar{g}$-continuous function and studied some its properties.

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1 Introduction

A space X equipped with two arbitrary topologies τ_1 and τ_2 is defined by Kelley [1] as the bitopological space in 1963 and denoted it by a triple (X, τ_1, τ_2) to generalize a topological space (X, τ). Every bitopological space (X, τ_1, τ_2) can be regarded as a topological space (X, τ) if $\tau_1 = \tau_2 = \tau$. A topological space occurs for every metric spaces but the bitopological spaces occurs for quasi-metric spaces. A subset A of a bitopological space (X, τ_1, τ_2) is called open if A is both τ_1-open and τ_2-open. The closed sets in topological spaces have many important properties. A closed subset of a compact space is compact; a closed subset of a normal space is normal; a closed subset of a complete uniform space is complete and a compact set in a Hausdorff space is closed, etc. The generalized closed sets or simply g-closed sets in a topological space were introduced and studied by Levine [2] in 1970. He defined a set A to be a generalized closed set in a topological space (X, τ) if its closure is contained in every open super set of A. In bitipological spaces, Fukutake [3] introduced and investigated the concept of g-closed sets in 1985. There were many different kind of generalized closed sets on topological spaces and on bitopological spaces introduced by different authors. In this paper, we introduce another kind of generalized closed set in the bitopological space and compare this with some of the corresponding generalized closed sets and then analyzed its properties.

2 Preliminaries

Throughout this paper, we represent X and Y as the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) on which no separation axioms are assumed unless otherwise stated. For a subset of A of X, τ_i-cl(A) denotes the closure of A and τ_i-int(A) denotes the interior of A, respectively with respective to the topology τ_i.

In the topological space (X, τ), we recall the following closure sets.

Definition 2.1. A subset A of a topological space (X, τ) is called a

1. Semi Closure [4] of A, denoted by scl(A), is defined to be the intersection of all semi closed sets containing A.
2. Regular Closure [5] of A, denoted by RCl(A), is defined to be the intersection of all regualr closed sets containing A.
3. $\alpha-$Closure [6] of A, denoted by αcl(A), is defined to be the intersection of all α-closed sets containing A.

In the topological space (X, τ), we recall the following open sets.

Definition 2.2. A subset A of a topological space (X, τ) is called a

1. semi open [7] if $A \subseteq \text{cl}(\text{int}(A))$.
2. α-open [8] if $A \subseteq \text{int}(\text{cl}(\text{int }(A)))$.
3. regular open [9] if $A = \text{int}(\text{cl }(A))$.
4. semi-preopen [10] if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$.

We also recall some generalized closed sets defined in a topological space (X, τ).

Definition 2.3. A subset A of a topological space (X, τ) is called a

1. g-closed [2] (generalized closed) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.
2. $sg-$closed [11] (semi generalized closed) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in τ.

3. $gsp-$closed [12] (generalized semi pre closed) if spcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ.
4. gsg-closed [13] (generalized semi generalized closed) if cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is semi generalized open in τ.
5. g-r-closed [14] (generalized regular closed) if Rcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is an open in τ.
6. δ-closed [15] if $A =$cl$_\delta(A)$ where cl$_\delta(A) = \{x \in X : \text{ int}(\text{cl}(U)) \cap A \neq \phi,\ U \in \tau \text{ and } x \in U\}$.
7. $r\delta$-closed [16] (regular δ closed) if $A =$cl$_\delta(A)$ whenever $A \subseteq U$ and U is a regular open in τ.
8. $b^\sharp$-closed [17] if $A =$cl(int $(A)) \cap$ int(cl(A)).
9. $rgb^\sharp$-closed [18] (regular generalized $b^\sharp$ closed) if $b^\sharp$cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular open in τ.
10. $\hat{g}$-closed [19] if cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi open in τ.
11. $\delta\hat{g}$-closed [20] if cl$_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$-open in τ.
12. strongly (gsp)*-closed [21] if cl(int$(A)) \subseteq U$ whenever $A \subseteq U$ and U is a gsp-open in τ.

Now we recall some generalized closed sets in a bitopological space (X, τ_1, τ_2).

Definition 2.4. A subset A of a bitopological space (X, τ_1, τ_2) is called a

1. $\tau_1\tau_2$-g-closed [22] ($\tau_1\tau_2$-generalized closed) if τ_2-cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.
2. $\tau_1\tau_2$-sg-closed [23] ($\tau_1\tau_2$-semi generalized closed) if τ_2-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open.
3. $\tau_1\tau_2$-gs-closed [24] ($\tau_1\tau_2$-generalized semi closed) if τ_2-scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- open.
4. $\tau_1\tau_2$-αg-closed [25] ($\tau_1\tau_2$-α-generalized closed) if τ_2-αcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- open.
5. $\tau_1\tau_2$-gα-closed [25] ($\tau_1\tau_2$-generalized α-closed) if τ_2-αcl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-α-open.
6. $\tau_1\tau_2$-$\hat{g}$-closed [24] if τ_2-cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1- semi open.

Definition 2.5. Let τ_1 and τ_2 be two topologies on a set X such that τ_1 is contained in τ_2. Then, the topology τ_1 is said to be a coarser (weaker or smaller) topology than τ_2.

3 Generalized $\tau_1\tau_2$-$\bar{g}$-Closed Sets

Definition 3.1. A subset A of a bitopological space (X, τ_1, τ_2) is called a $\tau_1\tau_2$-$\bar{g}$-closed if τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$.

Example 3.1. *Let* $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, X\}$, *and* $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$. *Then,* $\phi, \{c\}, \{a, c\}$, $\{b, c\}$ *and* X *are the* $\tau_1\tau_2$-$\bar{g}$*-closed sets in* (X, τ_1, τ_2).

Theorem 3.2. *The union of two* $\tau_1\tau_2$-$\bar{g}$*-closed sets is a* $\tau_1\tau_2$-$\bar{g}$*-closed set.*

Proof. Let A and B be two $\tau_1\tau_2$-$\bar{g}$-closed sets. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$ and τ_i-cl$(B) \subseteq U_i$ whenever $B \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Then, we have τ_i-cl$(A \cup B) \subseteq U_i$ whenever $(A \cup B) \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Therefore, $A \cup B$ is a $\tau_1\tau_2$-$\bar{g}$-closed set. □

The intersection of two $\tau_1\tau_2$-$\bar{g}$-closed sets need not be a $\tau_1\tau_2$-$\bar{g}$-closed set. This can be seen from the following example.

Example 3.3. *Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a\}, X\}$, and $\tau_2 = \{\phi, \{b\}, X\}$. If $A = \{a, b\}$ and $B = \{a, c\}$, then the sets A and B are $\tau_1\tau_2$-$\bar{g}$-closed; but, $A \cap B = \{a\}$ is not a $\tau_1\tau_2$-$\bar{g}$-closed set.*

Theorem 3.4. *A gsg-closed set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-closed set in (X, τ_1, τ_2).*

Proof. Let A be any gsg-closed set in both τ_1 and τ_2 and U_i be any open set in τ_i containing A for $i = 1, 2$ respectively. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is semi open in τ_i for $i = 1, 2$. Since any open set is a sg-open set, τ_i-cl$(A) \subseteq U_i$ for $i = 1, 2$. Therefore, A is $\tau_1\tau_2$-$\bar{g}$-closed. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.5. *Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{b\}, X\}$ and $\tau_2 = \{\phi, \{c\}, X\}$. The set $\{a\}$ is $\tau_1\tau_2$-$\bar{g}$-closed in $X = \{a, b, c\}$. But, it is not a gsg-closed set in both τ_1 and τ_2.*

Theorem 3.6. *A generalized regular closed set or g-r-closed set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-closed set in (X, τ_1, τ_2).*

Proof. Let A be any g-r-closed set in both τ_1 and τ_2. Then, τ_i-cl$(A) \subseteq \tau_i$-Rcl(A) for $i = 1, 2$. Hence A is a g-closed set in both τ_1 and τ_2 from the definition of generalized regular closed set, so, A is a $\tau_1\tau_2$-$\bar{g}$-closed set in (X, τ_1, τ_2). □

The converse of the above theorem need not be true as seen from the following example.

Example 3.7. *Let $X = \{a, b, c\}, \tau_1 = \{\phi, \{a, b\}, \{b, c\}, \{b\}, X\}$, and $\tau_2 = \{\phi, \{a, b\}, \{a, c\}, \{a\}, X\}$. The set $\{c\}$ is a $\tau_1\tau_2$-$\bar{g}$-closed. But, it is not a generalized regular closed set in both τ_1 and τ_2.*

Theorem 3.8. *Every $r\delta$-closed set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-closed set.*

Proof. Let A be $r\delta$-closed set in both τ_1 and τ_2. Then, $A =$cl$_\delta(A)$ whenever $A \subseteq U_i$ and U_i is a regular open in τ_i for $i = 1, 2$. So, cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is a open in τ_i for $i = 1, 2$ as every regular open set is open. Hence every $r\delta$-closed set in τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{b, c\}, X\}$; $\tau_2 = \{\phi, \{c\}, \{b, c\}, X\}$. and $A = \{b\}$. Hence the set A is a $\tau_1\tau_2$-$\bar{g}$-closed. But, it is not a $r\delta$-closed set in τ_1 and τ_2.*

Theorem 3.10. *If A is $\tau_1\tau_2$-$\bar{g}$-closed set, then A is rgb$^\sharp$-closed set in both τ_1 and τ_2.*

Proof. Let A be any $\tau_1\tau_2$-$\bar{g}$-closed set in X such that $A \subseteq U_i$ and U_i is regular open of τ_i for $i = 1, 2$ respectively. Hence A is g-closed set in (X, τ_1) and (X, τ_2) as every regular open set is open, U_i is open for $i = 1, 2$. Also, τ_i-b$^\sharp$cl$(A) \subseteq \tau_i$-cl$(A) \subseteq U_i$ for $i = 1, 2$. Therefore, τ_i-b$^\sharp$cl$(A) \subseteq U_i$ and U_i is regular open for $i = 1, 2$. Hence A is rgb$^\sharp$-closed set in τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}; \tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}$ and $A = \{a\}$. Hence the set A is a rgb$^\sharp$-closed set in both τ_1 and τ_2. But, it is not a $\tau_1\tau_2$-$\bar{g}$-closed set.*

Theorem 3.12. *Every $\delta\hat{g}$-closed set in both τ_1 and τ_2 is a $\tau_1\tau_2$-$\bar{g}$-closed set.*

Proof. Let A be an $\delta\hat{g}$-closed set in τ_1 and τ_2 and U_i is any open set containing A in (X, τ_i) for $i = 1, 2$, respectively. Since every open set is $\hat{g}$-open and A is $\delta\hat{g}$-closed set in τ_1 and τ_2, τ_i-$\text{cl}_\delta(A) \subseteq U_i$ for every subset A of X for $i = 1, 2$. Since τ_i-cl$(A) \subseteq \tau_i$-$\text{cl}_\delta(A) \subseteq U_i$, τ_i-cl$(A) \subseteq U_i$, and hence A is g-closed set in τ_1 and τ_2. Therefore, A is $\tau_1\tau_2$-$\bar{g}$-closed set. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.13. *Let* $X = \{a, b, c\}; \tau_1 = \{\phi, \{b\}, \{a, c\}, X\}$; $\tau_2 = \{\phi, \{c\}, \{a, b\}, X\}$ *and* $A = \{a\}$. *Hence the set* A *is a* $\tau_1\tau_2$-$\bar{g}$*-closed. But, it is not a* $\delta\hat{g}$*-closed set in* τ_1 *and* τ_2.

Theorem 3.14. *Every* $\tau_1\tau_2$-$\bar{g}$*-closed set is strongly (gsp)***-closed set in both* τ_1 *and* τ_2.

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is open in τ_i for $i = 1, 2$ respectively. Now, let $A \subseteq U_i$ and U_i be (gsp)-open in τ_i for $i = 1, 2$ respectively. Since every open set is a (gsp)-open set, we have τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is (gsp)-open in τ_i for $i = 1, 2$. But, τ_i-cl(int(A))$\subseteq \tau_i$-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is (gsp)-open in τ_i for $i = 1, 2$. Hence τ_i-cl(int(A))$\subseteq U_i$ whenever $A \subseteq U_i$ and U_i is (gsp)-open in (X, τ_i) for $i = 1, 2$. Therefore, A is strongly (gsp)*-closed set in τ_1 and τ_2. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.15. *Let* $X = \{a, b, c\}; \tau_1 = \{\phi, \{a\}, \{a, b\}, X\}; \tau_2 = \{\phi, \{c\}, \{b, c\}, X\}$ *and* $A = \{b\}$. *Hence the set* A *is a strongly (gsp)***-closed set in* τ_1 *and* τ_2*. But, it is not a* $\tau_1\tau_2$-$\bar{g}$*-closed.*

Theorem 3.16. *If* τ_1 *is coarser than* τ_2*, then every* $\tau_1\tau_2$-$\bar{g}$*-closed set is a* $\tau_1\tau_2$*-g-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Since τ_1 is coarser than τ_2, we have τ_2-cl$(A) \subseteq U$, whenever $A \subseteq U$, U is τ_1-open. Hence A is a $\tau_1\tau_2-$g-closed set. □

Theorem 3.17. *If* τ_1 *is coarser than* τ_2*, then every* $\tau_1\tau_2$-$\bar{g}$*-closed set is a* $\tau_1\tau_2$*-sg-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Since every open set is semi-open, U_1 is τ_1-semi open. Therefore, τ_1-scl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-semi open. Since τ_1 is coarser than τ_2, τ_2-scl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-semi open. Hence A is a $\tau_1\tau_2$-sg-closed set. □

Theorem 3.18. *If* τ_1 *is coarser than* τ_2*, then every* $\tau_1\tau_2$-$\bar{g}$*-closed set is a* $\tau_1\tau_2$*-gs-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i-cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Therefore, τ_1-scl$(A) \subseteq U_1$, whenever $A \subseteq U_1, U_1$ is τ_1-open. Since τ_1 is coarser than τ_2,
τ_2-scl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-open. Hence A is a $\tau_1\tau_2-$gs-closed set. □

Theorem 3.19. *If* τ_1 *is coarser than* τ_2*, then every* $\tau_1\tau_2$-$\bar{g}$*-closed set is a* $\tau_1\tau_2$-αg*-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i- cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Therefore, τ_1-α cl$(A) \subseteq U_1$, whenever $A \subseteq U_1$. Since τ_1 is coarser than τ_2, τ_2-α cl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-open. Hence A is a $\tau_1\tau_2$-αg-closed set. □

Theorem 3.20. *If* τ_1 *is coarser than* τ_2*, then every* $\tau_1\tau_2$-$\bar{g}$*-closed set is a* $\tau_1\tau_2$-$g\alpha$*-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_i- cl$(A) \subseteq U_i$ whenever $A \subseteq U_i$ and U_i is τ_i-open for each $i = 1, 2$. Therefore, τ_1-α cl$(A) \subseteq U_1$ whenever $A \subseteq U_1, U_1$ is τ_1-α-open as every open set is α-open. Since τ_1 is coarser than τ_2, τ_2-α cl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-α-open. Hence A is a $\tau_1\tau_2$-$g\alpha$-closed set. □

Theorem 3.21. *If τ_1 is coarser than τ_2, then every $\tau_1\tau_2$-$\bar{g}$-closed set is a $\tau_1\tau_2$-$\widehat{g}$-closed set.*

Proof. Let A be $\tau_1\tau_2$-$\bar{g}$-closed set. Then, τ_1-cl$(A) \subseteq U_1$, U_1 is τ_1-open. Since every open set is semi open, τ_1-cl$(A) \subseteq U_1$, whenever $A \subseteq U_1, U_1$ is τ_1-semi open. Since τ_1 is coarser than τ_2, τ_2-cl$(A) \subseteq U_1$, whenever $A \subseteq U_1$, U_1 is τ_1-semi open. Hence A is a $\tau_1\tau_2$-$\widehat{g}$-closed set. □

Theorem 3.22. *Let A be $\tau_1\tau_2$-$\bar{g}$-closed set and $A \subset B \subset \tau_i$-cl$(A)$ for $i = 1, 2$. Then, B is a $\tau_1\tau_2$-$\bar{g}$-closed set.*

Proof. Let $B \subseteq U$ where U is open in τ_1. Then, $A \subset B \subseteq U$. Since A is a $\tau_1\tau_2$-$\bar{g}$-closed set, A is g-closed in (X, τ_1) and (X, τ_2). Hence τ_1-cl$(A) \subseteq U$ whenever $A \subseteq U$. Since $B \subset \tau_1$-cl(A) and τ_1-cl(B) is the smallest closed set containing B, $B \subset \tau_1$-cl$(B) \subset \tau_1$-cl$(A) \subseteq U$. Hence τ_1-cl$(B) \subset U$ whenever $B \subseteq U$. Hence B is g-closed in (X, τ_1). Similarly, we can show that B is g-closed in (X, τ_2). □

Definition 3.2. A function f from spaces (X, τ_1, τ_2) into (Y, σ_1, σ_2) is called $\tau_1\tau_2 - \bar{g}-$continuous if $f^{-1}(V)$ is $\tau_1\tau_2 - \bar{g}-$closed set in X for each σ_i-closed set V in Y.

Example 3.23. *Let $X = \{a, b, c\} = Y; \tau_1 = \{\phi, \{a, b\}, X\}$; $\tau_2 = \{\phi, \{b\}, \{a, c\}, X\}; \sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ defined by $f(a) = a$ is $\tau_1\tau_2 - \bar{g}-$continuous mapping.*

Theorem 3.24. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $\tau_1\tau_2 - \bar{g}-$continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \rho_1, \rho_2)$ is continuous, then $g \circ f$ is $\tau_1\tau_2 - \bar{g}-$continuous.*

Proof. Let A be ρ_i- closed set in Z. Since g is continuous, $g^{-1}(A)$ is σ_i-closed in Y. Since f is $\tau_1\tau_2 - \bar{g}-$continuous, $f^{-1}(g^{-1}(A))$ is $\tau_1\tau_2 - \bar{g}-$closed in X. Hence $g \circ f$ is $\tau_1\tau_2 - \bar{g}-$continuous. □

4 Conclusion

In this paper, $\tau_1\tau_2$-$\bar{g}$-closed sets were introduced in the bitopological spaces and their properties were studied. Further, their properties were compared with some of the corresponding generalized closed sets in the general topological spaces and bitopological spaces.

Competing Interests

Authors have declared that no competing interests exist.

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