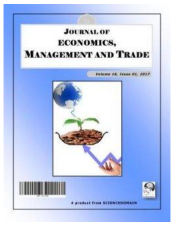

Journal of Economics, Management and Trade

21(9): 1-12, 2018; Article no.JEMT.43730
ISSN: 2456-9216
(Past name: British Journal of Economics, Management & Trade, Past ISSN: 2278-098X)

Tax Audit by Government, and Optimal Air Pollution Tax Rate

Yu-Kun Wang[1*]

[1]Department of Business Administration, National Taipei University of Business, No.321, Sec. 1, Jinan Rd., Zhongzheng District, Taipei City 100, Taiwan.

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/JEMT/2018/43730
Editor(s):
(1) Dr. Alfredo Jimenez Palmero, Kedge Business School, France.
(2) Dr. John M. Polimeni, Associate Professor of Economics, Albany College of Pharmacy & Health Sciences, New York, USA.
Reviewers:
(1) Somnath Das, Rabindra Mahavidyalaya, Champadanga, India.
(2) Mahfoudh Hussein Mgammal, Amran University, Yemen.
(3) J. Ramola Premalatha, Vellore Institute of Technology, India.
Complete Peer review History: http://www.sciencedomain.org/review-history/26298

Original Research Article

Received 23 June 2018
Accepted 13 September 2018
Published 19 September 2018

ABSTRACT

This paper uses two-stage game theory to analyze the relationship between tax evaders and government regarding air pollution emissions. The production function of a firm in a perfectly competitive market was first considered. Then, the backward induction method and the Cramer rule method to determine the optimal subgame perfect equilibrium in the two-stage game and investigate the relationship between firms' tax evasion behaviour and tax variables. This study discovered that the stronger the spillover effect on firms engaging in air pollution control, the higher the tax rate levied by government should be. When firms are in a perfectly competitive market and the financial policy instruments (i.e., air pollution tax and subsidy rate) are known, the conditions for economic stability can be established.

Keywords: Tax evasion; backward induction; Cramer rule; air pollution tax.

JFL Classification: H26, P43, P35.

**Corresponding author: E-mail: W550109@ntub.edu.tw;*

1. INTRODUCTION

Tax erosion occurs for various reasons. Some articles have argued that it is caused by high tax rates imposed by governments, which entices taxpayers to report an income that is considerably lower than their actual income (i.e., concealed income); some have remarked that high tax rates prompt taxpayers to shift their operations from on-the-ground markets to underground markets, which increases tax evasion behaviour; and others have argued that when concealment costs are lower than the probability of getting caught multiplied by the financial penalty of the concealment, tax evasion behaviour will likely ensue, increasing tax erosion. [1]. Firm-related tax evasion behavior involves other factors, including the external costs incurred by firms' pollution emission, companies' pollution-related tax evasion, firms' expected utility arising from uncertainty in the probability of being caught for evading taxes, and the effect of other firms' tax evasion behavior on representative firms' decisions to engage in tax evasion behavior. These topics were explored in this study.

2. LITERATURE REVIEW

A growing literature has conducted the behaviour between taxpayers and the government. Wang and Conant [2] and Yaniv [3] show that how firms balance production output and tax evasion to maximize their expected utility. In practice, a firm may produce pollution during the production of their products and/or services, which creates an external cost to the environment. In those studies, Baron [4] and Wu and Yen [5] assumed that the utility function of a representative firm is affected by both its profit and corporate social responsibility. This paper adopts the perspective of Baron [4] and Wu and Yen [5] (i.e., that pollution may be emitted during a firm's product and/or service production) and the model introduced by Hsu and Tsui [6] (i.e., the optimal air pollution tax and subsidy rate model to be used by government while considering the spillover effect). Other related literature, Slemrod and Yitzhaki [7] and Yitzhaki [8] proposed the probability that a representative firm is caught for tax evasion to express an incremental function representing concealed profit. Wu and Yen (2011) proposed the concept of firm's size of tax evasion by representing it in the form of "actual profits" and "reported profits."

3. BASE MODEL

Because tax evaders and a government are in a game-based relationship, this paper employs the W-C-Y model. The model is employed to analyze the tax game between firms and government. A firm's actual profit can be represented by the following definitional equation:

$$\pi(Q) = R(Q) - C(Q) \tag{1}$$

Where, *R* is the firm's total income, *C* is the firm's total costs, *Q* is the production output, and *R* and *C* are functions of *Q*. In addition, $R^{'}(Q) > 0$, $R^{''}(Q) < 0$, $C^{'}(Q) > 0$, and $C^{''}(Q) > 0$, indicating that the second-order conditions of the profit function are a decreasing concave function and that the second-order conditions of the cost function are an increasing convex function.

3.1 Model Assumptions

This paper assumes that *m* is the proportional tax levied by a government, that $0 < m < 1$ and that a firm confronted with proportional tax may evade taxes by reporting a profit lower than their actual profit.

Next, this paper assumes that *Z* is the profit declared by the firm; $Z \le \pi$ (i.e., the profit declared by the firm is less than or equal to its actual profit); the government penalty rate is τ (Where, $\tau > 1$); the government performs random profit-checking; and *q* is the probability that the firm gets caught (an exogenous variable). In contrast to previous literature, the concealment cost of a firm is defined as a definite integral with a lower limit of 0: $\int_0^s h(\mathrm{s})ds = H(\mathrm{s})$, where *s* is the effort made by the firm regardless of whether it is caught for tax evasion ($1 \ge s \ge 0$). The firm emits pollution during product and/or service production; the firm's expected utility is unaffected by whether its pollution emissions fall within the standard pollution emissions ($\bar{Q}$) specified by the country's environmental protection unit, and $\tilde{Q} = Q - \bar{Q}$ indicates the firm's illegal pollution emissions. Unlike previous articles, this paper accounts for A^{d}, the negative psychological effect of *h* percent of firms evading taxes ($0 \le h \le 1$) on the representative firms in a society; thus, the fewer firms that engage in tax evasion, the more moral pressure is experienced by representative firms when they evade taxes. In addition, this paper adds representative-tax-evader concealment cost functions for further analyses.

3.2 Taxpayers' Optimal Solutions in Response to Taxes

The first stage involves government formulating the most appropriate policy instruments (e.g., tax rates, penalties, and maximum pollution emission threshold for product and service production processes). The second stage entails the taxpayers' optimal solutions given the policy instruments employed by the government. A representative firm's expected utility in production output and tax evasion can be represented as follows:

i) A representative firm's after-tax profit when it successfully evades taxes can be represented by the following formula:

$$\pi^{\mathrm{UA}} = (1-m)\times\pi + m\times(\pi - Z) - H(s) \quad (2)$$

ii) A representative firm's after-tax profit when it is caught for tax evasion can be represented by the following formula:

$$\pi^{\mathrm{A}} = (1-m)\times\pi - \tau\times m\times(\pi - Z) - H(s) \quad (3)$$

This paper proposes the probability (*q*) that a representative firm is caught for tax evasion to express an incremental function representing concealed profit.

The expected utility function for a representative firm caught or not caught evading taxes can thus be expressed as follows:

$$EU(Q,Z) = (1-q)\times U(\pi^{\mathrm{UA}}) + q\times U(\pi^{\mathrm{A}}) - A^d\times h - b\times V(E(Q)-\bar{Q}) \quad (4)$$

Where, $V(E(Q)-\bar{Q})$ is the negative effect when a firm's pollution emission exceeds $\bar{Q}$, and *b* is the firm's level of concern for the external cost created as a result of the pollution emissions generated during product and service production. The decision to be made by a representative firm concerning its production output and tax evasion behaviour can be expressed as follows:

$$Max \quad U(Q,Z) = (1-q)\times U(\pi^{\mathrm{UA}}) + q\times U(\pi^{\mathrm{A}}) - A^d\times h - b\times V(E(Q)-\bar{Q}) \quad (5)$$

s.t. $\pi^{\mathrm{UA}} = (1-m)\times\pi + m\times(\pi - Z) - H(s)$

$$\pi^{\mathrm{A}} = (1-m)\times\pi - \tau\times m\times(\pi - Z) - H(s)$$

The first-order conditions of this decision equation can be represented as follows:

$$U_Z = m\times\left\langle \tau\times q\times U'(\pi^{\mathrm{A}}) - (1-q)\times U'(\pi^{\mathrm{UA}})\right\rangle = 0 \quad (6)$$

$$U_Q = (R'-C')\times\begin{bmatrix}(1-q)\times U'(\pi^{\mathrm{UA}}) + q\times(1-m) \\ \times U'(\pi^{\mathrm{A}}) - \tau\times m\times q\times U'(\pi^{\mathrm{A}})\end{bmatrix} - b\times V'\times E' = 0 \quad (7)$$

Where, U_Z and U_Q represent a firm's marginal expected utility in reported profit and in its production output, respectively. Solving (6) and (7) yields a firm's optimal reported profit (Z^*) and production output (Q^*). This paper investigates the second-order conditions of a firm's decision-making problems, where

$\mathrm{U}_{ZZ} < 0$; $\mathrm{U}_{QQ} < 0$; and $H = U_{ZZ}\times U_{QQ} - (U_{ZQ})^2 > 0$.

The perspective of Wu and Yen (2011) and is adopted, and it is assumed that a representative firm is not concerned with the external cost of its pollution emissions during product and service production; that is, $b = 0$. Substituting this into (7) gives

$$U_Q = (R'-C')\times\begin{bmatrix}(1-q)\times U'(\pi^{\mathrm{UA}}) + q\times(1-m) \\ \times U'(\pi^{\mathrm{A}}) - \tau\times m\times q\times U'(\pi^{\mathrm{A}})\end{bmatrix} = 0 \quad (8)$$

Equation (8) reveals that when marginal revenue equals marginal cost, a firm's reported profit and its production output are neutral, a finding that supports the Wang–Conant (W–C) proposition.

Applying the first-order conditions to (6) enables obtaining the following equation:

$$U_{ZQ} = m\times(R'-C')\times\begin{bmatrix}q\times\tau\times U''(\pi^{A})\times\{(1-m)-\tau\times m\} \\ -U''(\pi^{UA})\times(1-q)\end{bmatrix} \quad (9)$$

Equation (6) shows that $\tau \times q \times U'(\pi^{\mathrm{A}}) = (1-q) \times U'(\pi^{\mathrm{UA}})$ which can be substituted into (9) to obtain (10).

$$U_{ZQ} = m \times (R' - C') \times q \times \tau \times U'(\pi^A) \times \left\{ r(\pi^{UA}) - r(\pi^A) \times (1 - m - \tau \times m) \right\} \tag{10}$$

Where, $r(\pi^{\mathrm{A}}) = \frac{-\mathrm{U}''(\pi^{\mathrm{A}})}{\mathrm{U}'(\pi^{\mathrm{A}})}$ and $r(\pi^{\mathrm{UA}}) = \frac{-\mathrm{U}''(\pi^{\mathrm{UA}})}{\mathrm{U}'(\pi^{\mathrm{UA}})}$.

i).Equation (10) demonstrates that when $b = 0$ and a firm is in equilibrium, then $(R' - C') = 0$, which suggests that marginal revenue equals marginal costs and that the firm's reported profit and production output have become neutral, supporting the W–C proposition (1988).

ii).Equations (7) reveals that when $b > 0$, $(R' - C') = 0$ is not the optimal production output decision. Under this scenario, $(R' - C') > 0$; hence, (10) shows that when a representative firm's coefficient of absolute risk aversion decreases, or $r(\pi^A) > r(\pi^{UA})$, $\frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m$ ensures that $U_{ZQ} > 0$, meaning that a firm's increased production output leads to increased reported income and decreased tax evasion. Equation (10) thus carries the following critical economic implication: when a firm emits more pollution than the pollution emissions standard set by government, the psychological anxiety experienced by the firm prompts it to increase its production and thus its reported profit, reducing tax evasion. However, for this to occur, the following premise must be true: "the ratio between the coefficient of absolute risk aversion when a firm is not caught for tax evasion and that of absolute risk aversion ratio when a firm is caught for tax evasion" must be greater than "the tax to be paid by the firm for each dollar earned minus the tax to be paid for each dollar earned then multiplied by the tax evasion penalty."

3.3 Comparative Static Analyses of Firm Production Output and Reported Profits

This paper uses the implicit function theorem to determine the relationships of governmental policy instrument variables ($\Omega \in \{\tau, q, m\}$) with optimal production output and reported profit, where $\tau,\ q,\ m$ denotes the penalty, probability that a firm is caught for tax evasion, and tax rate, respectively. Thus,

$$\frac{\partial Z^*}{\partial \Omega} = \frac{U_{ZQ} \times U_{Q\Omega} - U_{Z\Omega} \times U_{QQ}}{H} \tag{11}$$

$$\frac{\partial Q^*}{\partial \Omega} = \frac{U_{Z\Omega} \times U_{QZ} - U_{ZZ} \times U_{Q\Omega}}{H} \tag{12}$$

Given that $H = U_{ZZ} \times U_{QQ} - (U_{ZQ})^2 > 0$, whether (11) and (12) are greater or less than zero is determined by the sign of the numerator *H*.

i) When, $b = 0$, (8) implies that $U_{ZQ} = U_{QZ} = U_{Q\Omega} = 0$. Accordingly, (11) and (12) can be expressed as follows:

$$\frac{\partial Z^*}{\partial \Omega} = \frac{-U_{Z\Omega} \times U_{QQ}}{H} \tag{13}$$

$$\frac{\partial Q^*}{\partial \Omega} = 0 \tag{14}$$

Equation (14) shows that when $b = 0$, changes in policy instruments (e.g., tax rates, penalties, and probabilities that firms are caught for tax evasion) will not change a firm's production output model. Given that $U_{QQ} < 0$ and that $H > 0$, whether (13) is greater or less than 0 is determined by whether $U_{Z\Omega}$ is greater or less than 0.

Substituting the penalty τ, the probability that a firm is caught for tax evasion *q*, and the tax rate *m* into (6) yields the following equations:

$$U_{Z\tau} = m \times \left[q \times U'(\pi^A) - m \times (\pi - Z) \times \tau \times q \times U''(\pi^A) \right] \tag{15}$$

Equation (15) reveals that $U_{Z\tau} > 0$, indicating that a firm's reported profit and tax evasion penalty are in a complementary relationship.

$$U_{Zq} = m \times \left[\tau \times U'(\pi^{\mathrm{A}}) + U'(\pi^{\mathrm{UA}}) \right] > 0 \tag{16}$$

Equation (16) shows that a firm's reported profit (Z) and the probability that a firm is caught for tax evasion are also in a complementary relationship. Equation (6) shows that $\tau \times q \times U'(\pi^{\mathrm{A}}) = (1-q) \times U'(\pi^{\mathrm{UA}})$. Substituting the coefficient of absolute risk aversion $[r(\pi^i) = -U''(\pi^{\mathrm{i}})/U'(\pi^{\mathrm{i}}),\ i = A,\ UA]$ into (17) yields (18):

$$U_{Zm} = \tau \times q \times U'(\pi^{\mathrm{A}}) - (1-q) \times U'(\pi^{\mathrm{UA}}) - (\pi + \tau \times \pi) \times m \times \tau \times q \times U''(\pi^{\mathrm{A}}) + Z \times m \times (1-q) \times U''(\pi^{\mathrm{UA}}) \tag{17}$$

$$U_{Zm} = m \times (1-q) \times U'(\pi^{\mathrm{UA}}) \times \left[(\pi + \tau \times \pi) \times r(\pi^{\mathrm{A}}) - Z \times r(\pi^{\mathrm{UA}}) \right] \tag{18}$$

Because $r(\pi^{\mathrm{A}}) > r(\pi^{\mathrm{UA}})$ and $(\pi + \tau \times \pi) > Z$, $U_{Zm} > 0$. Equations (15), (16), and (18) reveal that the relationship between changes in policy instrument variables and a representative firm's optimal reported profit is denoted by $U_{Z\Omega} > 0$; that is,

$$\frac{\partial Z^*}{\partial \Omega} = \frac{-U_{Z\Omega} \times U_{QQ}}{H} > 0$$

Where, $\begin{bmatrix} U_{ZZ} & U_{ZQ} \\ U_{QZ} & U_{QQ} \end{bmatrix} \begin{bmatrix} dZ \\ dQ \end{bmatrix} = \begin{bmatrix} -U_{Z\Omega} \\ -U_{Q\Omega} \end{bmatrix}$, $H = \begin{bmatrix} U_{ZZ} & U_{ZQ} \\ U_{QZ} & U_{QQ} \end{bmatrix} > 0$.

This indicates that when the government increases penalties, tax rates, or the probability of firms being caught for tax evasion, firms increase their reported profits.

ii) If $b > 0$, a firm will be psychologically anxious about emitting more pollution than the pollution emissions standard set by the government; (10) shows that when $\dfrac{r(\pi^{UA})}{r(\pi^{A})} > 1 - m - \tau \times m$, $U_{ZQ} > 0$.

In addition, (7) shows that $U_{Q\Omega} \neq 0$. The calculations for the two variables are described next.

First, the effect of penalties on production output can be obtained by substituting τ into (7) as a first-order condition:

$$U_{Q\tau} = -(1-m) \times (R' - C') \times q \times m \times (\pi - Z) - m \times (R' - C') \times q \times U'(\pi^{\mathrm{A}}) + (R' - C') \times \tau \times m^2 \times q \times U''(\pi^{\mathrm{A}}) \times (\pi - Z)$$

$$\begin{cases} \text{if } (R' - C') < 0, \text{ then } U_{Q\tau} > 0 \\ \text{if } (R' - C') > 0, \text{ then } U_{Q\tau} < 0 \end{cases} \tag{19}$$

Equation (19) demonstrates that if the tax evaded by a firm is the tax base for calculation of the subsequent penalty and $(R' - C') > 0$, an increase in penalty reduces a firm's production output ($U_{Q\tau} < 0$). However, if $(R' - C') < 0$, then an increase in penalty increases a firm's output. In practice, $(R' - C') < 0$ rarely occurs in the long term.

Second, the effect of the probability of the government performing tax checking (q) on production output (U_{Qq}) is calculated by substituting q into (7) as a first-order condition:

$$\begin{aligned} U_{Qq} &= -(R' - C') \times U'(\pi^{\mathrm{UA}}) + (1-m) \times U'(\pi^{\mathrm{A}}) \times (R' - C') - \tau \times m \times U'(\pi^{\mathrm{A}}) \times (R' - C') \\ &= -(R' - C') \times U'(\pi^{\mathrm{UA}}) + U'(\pi^{\mathrm{A}}) \times (R' - C') \times (1 - m - \tau \times m) \end{aligned} \tag{20}$$

$$\begin{cases} \text{If } (R' - C') > 0 \text{ and } m \geq \dfrac{1}{1+\tau}, \text{ then } U_{Qq} < 0 \\ \text{If } (R' - C') > 0 \text{ and } m < \dfrac{1}{1+\tau}, \text{ then } U_{Qq} \text{ may be positive or negative} \end{cases}$$

Equation (20) reveals that the tax evaded by a firm is the tax base for calculation of the subsequent penalty; when $(R^{'}-C^{'})>0$ and $m \geq \frac{1}{1+\tau}$, an increase in the probability of the government performing tax audit decreases a firm's output capacity ($U_{Q\tau}<0$). However, when $m < \frac{1}{1+\tau}$, an increase in the probability of the government performing tax audit may increase or decrease a firm's output capacity.

Third, the effect of tax rate on production output (U_{Qm}) is calculated by substituting m into (7) as a first-order condition:

$$
\begin{aligned}
U_{Qm} =& -Z\times(1-q)\times(R^{'}-C^{'})-(R^{'}-C^{'})\times q\times U^{'}(\pi^{A})- \\
& (\pi+\tau\times\pi-\tau\times Z)\times q\times(1-m)\times U^{''}(\pi^{A})\times(R^{'}-C^{'})-(R^{'}-C^{'})\times\tau\times q\times U^{'}(\pi^{A}) \\
& +\tau\times q\times m\times U^{''}(\pi^{A})\times(R^{'}-C^{'})\times(\pi+\tau\times\pi-\tau\times Z)
\end{aligned}
\quad (21)
$$

Equation (21) can be further modified into the following:

$$
\begin{aligned}
U_{Qm} =& -Z\times(1-q)\times(R^{'}-C^{'})-(R^{'}-C^{'})\times q\times U^{'}(\pi^{A})-(R^{'}-C^{'})\times\tau\times q\times U^{'}(\pi^{A}) \\
& -(\pi+\tau\times\pi-\tau\times Z)\times q\times U^{''}(\pi^{A})\times(R^{'}-C^{'})\times(1-m-\tau\times m)
\end{aligned}
\quad (22)
$$

Equation (22) shows that when $(R^{'}-C^{'})>0$ and $m \geq \frac{1}{1+\tau}$, then $U_{Qm}<0$; conversely, when $(R^{'}-C^{'})>0$,and $m < \frac{1}{1+\tau}$, then U_{Qm} may be positive or negative.

Equation (22) thus reveals that the condition $(R^{'}-C^{'})>0$ alone cannot guarantee whether an increase in tax rate has a positive or negative effect on production output.

This paper uses the implicit function theorem, (6), and (7) to identify the relationships of minimal changes in governmental policy instrument variables with a firm's optimal reported profit and production output, as represented by the following:

$$\frac{\partial Z^{*}}{\partial\Omega}=\frac{U_{ZQ}\times U_{Q\Omega}-U_{Z\Omega}\times U_{QQ}}{H} \quad (23)$$

$$\frac{\partial Q^{*}}{\partial\Omega}=\frac{U_{ZQ}\times U_{QZ}-U_{ZZ}\times U_{Q\Omega}}{H} \quad (24)$$

Given that $H>0$, whether (23) and (24) are greater than, equal to, or less than zero is determined by the sign of the numerator *H*.

The relationship between a firm's pollution emissions and its social responsibility is next explored.

iii) The relationship between a firm's social responsibility and its reported profit:

When $b=0$, $U_{ZQ}=U_{QZ}=U_{Q\Omega}=0$, indicating that marginal revenue equals marginal cost and that a firm's reported profit and its production output have become neutral, supporting the W–C proposition; that is,

$$\frac{\partial Z^{*}}{\partial\Omega}=\frac{-U_{Z\Omega}\times U_{QQ}}{H} \quad (25)$$

$$\frac{\partial Q^{*}}{\partial\Omega}=0 \quad (26)$$

Equation (10) shows that when $b>0$, the representative firm's coefficient of absolute risk aversion is decreased, or $r(\pi^{A})>r(\pi^{UA})$. Therefore, $\frac{r(\pi^{UA})}{r(\pi^{A})}>1-m-\tau\times m$ must be true to ensure that $U_{ZQ}>0$. This signifies that an increase in a firm's production output will increase its reported profit and reduce its tax evasion behaviour. In addition, (19)–(22) show that when $b>0$, increases in penalties, the probability of tax audit by the government, and the tax rate do not guarantee that firms' tax evasion behaviour will be effectively deterred.

iv) When a firm has a higher social responsibility (*b*), the amount of profit reported by the firm will be higher only if $\frac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}<1-m-\tau\times m$; this subsequently lowers the firm's tax evasion behaviour, as shown in (27).

Given that $U_Z=m\times\left\langle\tau\times q\times U'(\pi^{\mathrm{A}})-(1-q)\times U'(\pi^{\mathrm{UA}})\right\rangle=0$ (6) $U_{\mathrm{Zb}}=0$, and given that $U_Q=(R'-C')\times\begin{bmatrix}(1-q)\times U'(\pi^{\mathrm{UA}})+q\times(1-m)\times U'(\pi^{\mathrm{A}})\\-\tau\times m\times q\times U'(\pi^{\mathrm{A}})\end{bmatrix}-b\times V'\times E'=0$

$U_{\mathrm{Qb}}=-V'\times E'<0$, the following equation can be derived by using $U_{\mathrm{QQ}}<0$, $U_{\mathrm{ZZ}}<0$, and $H>0$:

$$\frac{\partial Z^*}{\partial b}=\frac{U_{ZQ}\times U_{Qb}-U_{Zb}\times U_{QQ}}{H},$$

$$\begin{cases}\text{when } \dfrac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}>1-m-\tau\times m,\ U_{\mathrm{ZQ}}>0 \ \text{ and } \dfrac{\partial Z^*}{\partial b}<0\\ \text{when } \dfrac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}<1-m-\tau\times m,\ U_{\mathrm{ZQ}}<0 \ \text{ and } \dfrac{\partial Z^*}{\partial b}>0\end{cases} \tag{27}$$

Equation (27) reveals that when $\frac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}<1-m-\tau\times m$, an increase in a firm's social responsibility will increase its reported profit, reducing its tax evasion behaviour. However, when $\frac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}>1-m-\tau\times m$, an increase in a firm's social responsibility will lower its reported profit, increasing its tax evasion behaviour.

v) Regardless of the tax rate, an increase in a firm's social responsibility will diminish its output capacity:

$$\frac{\partial Q^*}{\partial b}=\frac{U_{Zb}\times U_{QZ}-U_{ZZ}\times U_{Qb}}{H}=\frac{-U_{ZZ}\times U_{Qb}}{H} \tag{28}$$

Given that $U_{\mathrm{Qb}}<0$, $U_{Zb}=0$, $U_{ZZ}<0$, and $H>0$, $\frac{\partial Q^*}{\partial b}<0$.

Equation (28) demonstrates that when a firm has a higher social responsibility ($b>0$), an increase in the firm's production output will result in an increase in the negative effect. Thus, when a firm has a higher social responsibility, it will lower its output capacity to reduce the negative effect.

vi) The effect of increased pollution emission standards ($\bar{Q}$) on a firm's production output and reported profit is explored. Equation (6) shows that $U_{Z\bar{Q}}=0$. When $\frac{r(\pi^{\mathrm{UA}})}{r(\pi^{\mathrm{A}})}>1-m-\tau\times m$, $U_{\mathrm{ZQ}}>0$, $H>0$, $\frac{\partial Z^*}{\partial\bar{Q}}>0$, and $U_{Q\bar{Q}}=V''\times E'\times b>0$ (as shown in (7)), (29) is used to determine that $\frac{\partial Z^*}{\partial\bar{Q}}>0$. Accordingly, an increase in pollution emission standards will facilitate raising a firm's reported profit:

$$\begin{bmatrix}U_{ZZ} & U_{ZO}\\ U_{QZ} & U_{QQ}\end{bmatrix}\begin{bmatrix}dZ\\ dQ\end{bmatrix}=\begin{bmatrix}-U_{Z\bar{Q}}\\ -U_{Q\bar{Q}}\end{bmatrix}$$

$$\frac{\partial Z^*}{\partial\bar{Q}}=\frac{U_{ZQ}\times U_{Q\bar{Q}}-U_{Z\bar{Q}}\times U_{QQ}}{H}>0 \tag{29}$$

vii) Similarly, given that $U_{Z\bar{Q}}=0$, $U_{\mathrm{ZZ}}<0$, $U_{Q\bar{Q}}>0$, and $H>0$, the following equation can be derived:

$$\frac{\partial Q^*}{\partial\bar{Q}}=\frac{U_{QZ}\times U_{Z\bar{Q}}-U_{Q\bar{Q}}\times U_{ZZ}}{H}>0 \tag{30}$$

which reveals that $\frac{\partial Q^*}{\partial\bar{Q}}>0$ and that an increase in pollution emission standards set by government can help increase a firm's output capacity and thereby its production output.

4. SUBGAME PERFECT EQUILIBRIUM BETWEEN TAXPAYERS AND GOVERNMENT

This paper explores the production function of representative firms (an exogenous variable) and their tax evasion behavior in addition to the definitional equations of and causal relationships between firms' tax evasion behavior and the tax rates and subsidy rates offered by the government to domestic firms engaging in air pollution control to reduce external costs caused by production. To analyze the two-stage game describing the interaction between tax evaders and government. In Stage 1 of the game, firms identify their optimal production output and air pollution emissions, whereas in Stage 2, the government substitutes the spillover effect and firms' production output and air pollution emissions into the social welfare function to determine the optimal tax subsidy rates to be offered to firms engaging in air pollution control. In the two-stage game settings, when all market participants play the tax game while knowing all information, backward induction can be used to solve the subgame perfect equilibrium between firms and government. Calculations can then be made to identify the optimal production output and air pollution emissions (i.e., Stage 1 solutions) and determine the optimal tax and subsidy rates to be offered by the government to firms engaging in air pollution control (i.e., Stage 2).

4.1 Model Assumptions

Assuming that a firm's revenue function is $R(\mathrm{Q}_{\mathrm{i}})$; m is the air pollution tax levied by government when a firm emits air pollution that exceeds the emission standards; D_0 is the air pollution allowed to be emitted during the production process, as stipulated by the environmental protection agency; $C_i^{max}(Q_i, \overset{\exists}{D}_i)$ is the production cost for air pollution emitted by the i^{th} firm when the firm does not engage in air pollution control; $\mathrm{C}_i(Q_i, \overset{\otimes}{D}_i)$ is the i^{th} firm's production cost if it engages in air pollution control; and $\overset{\otimes}{D}_i$ is the air pollution emitted by the i^{th} firm when the firm does engage in air pollution control. $\frac{\partial \mathrm{C}_i}{\partial Q_i} > 0$ and $\frac{\partial \mathrm{C}_i}{\partial \overset{\otimes}{D}_i} < 0$, where $\overset{\otimes}{D}_i < \overset{\exists}{D}_i$. $\mathrm{C}_i(Q_i, \overset{\otimes}{D}_i) - C_i^{max}(Q_i, \overset{\exists}{D}_i)$ denotes the cost to a firm engaging in air pollution prevention and control. $\mathrm{g}_i(\overset{\otimes}{D}_j)$ is defined as the spillover effect of the j^{th} firm's air pollution emissions on the i^{th} firm, where the spillover effect causes the i^{th} firm to invest in air pollution control (where the cost is $\mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[\mathrm{C}_i(Q_i, \overset{\otimes}{D}_i) - C_i^{max}(Q_i, \overset{\exists}{D}_i)\right]$).

In particular, $\mathrm{g}_i(\overset{\otimes}{D}_j) = 1$ indicates that the j^{th} firm has no spillover effect on the i^{th} firm. Note that $0 < \mathrm{g}_i(\overset{\otimes}{D}_j) \leq 1$ and $\mathrm{g}_i(\overset{\otimes}{D}_j) > 0$; the smaller $\mathrm{g}_i(\overset{\otimes}{D}_j)$ is, the stronger the spillover effect becomes. R_{Q} is the marginal revenue of the i^{th} firm for producing one additional unit of product or service; $\mathrm{C}_{\mathrm{Q}}^{\max}$ and C_{Q} are the marginal cost of the i^{th} firm for producing one additional unit of product or service if they do not and do engage in air pollution control, respectively; a_u is the subsidy rate offered by the government for a firm that engages in air pollution control; v_i is the tax erosion rate; and $M_{\mathrm{h}}(\overset{\otimes}{D}_i - D_0)$ is a firm's expected penalty when its air pollution emissions exceed the standards (Harford) [9].

The model introduced in this paper assumes that the i^{th} firm's air pollution concealment cost is minimal (close to zero) and that its profit equals its revenue plus the government's air pollution control subsidy minus its production costs, air pollution control costs, air pollution tax levied by the government, and expected penalty for air pollution emissions that exceed the standards. Accordingly, the objective function that maximizes profit for the firm is as follows:

$$\underset{Q_i \overset{\otimes}{D}_i}{Max}\,\pi_i = R(Q_i) - \mathrm{C}_i(Q_i, \overset{\otimes}{D}_i) + a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[\mathrm{C}_i(Q_i, \overset{\otimes}{D}_i) - C_i^{max}(Q_i, \overset{\exists}{D}_i)\right] - M_{\mathrm{h}}(\overset{\otimes}{D}_i - D_0) - m \times \overset{\otimes}{D}_i \times (1 - v_i) \tag{31}$$

4.2 Optimal Decisions for Representative Firms

Based on the aforementioned assumptions, given that the financial policy instruments (i.e., air pollution tax and subsidy rate) are known, and assuming that the i^{th} firm is in a perfectly competitive market, the first-order derivative function describing production output and air pollution emission quantity can be solved using (31):

$$\pi_{Qi} \equiv \frac{\partial \pi}{\partial Q} = R_{\mathrm{Q}} - C_{\mathrm{Q}} + a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max} \right] = 0 \tag{32}$$

$$\pi_{\overset{\otimes}{D}_i} \equiv \frac{\partial \pi}{\partial \overset{\otimes}{D}} = -C_{\overset{\otimes}{D}} + a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times C_{\overset{\otimes}{D}} - M_{\mathrm{h}}' - m \times (1 - v_i) = 0 \tag{33}$$

Through using conventional model settings, the second-order conditional derivative functions for profit maximization can be formulated:

$$\frac{\partial^2 \pi}{\partial Q_i^2} = R_{\mathrm{QQ}} - C_{\mathrm{QQ}} + a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{QQ}} - C_{\mathrm{QQ}}^{\max} \right] < 0$$

and $\frac{\partial^2 \pi}{\partial \overset{\otimes}{D}_i^2} = C_{\overset{\otimes}{D}\overset{\otimes}{D}} \times \left[a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1 \right] - M_{\mathrm{h}}'' < 0$

Through taking the total derivative of (32) and (33) simultaneously, the following equation can be derived:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} dQ_i \\ d\overset{\otimes}{D}_i \end{bmatrix} =$$

$$\begin{bmatrix} -\mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{QQ}} - C_{\mathrm{QQ}}^{\max} \right] \times da_u - a_u \times \left[C_{\mathrm{QQ}} - C_{\mathrm{QQ}}^{\max} \right] \times \mathrm{g}' \, d\overset{\otimes}{D}_j \\ d(m \times (1 - v)) - \mathrm{g}_i(\overset{\otimes}{D}_j) \times C_{\overset{\otimes}{D}} \times da_u - a_u \times C_{\overset{\otimes}{D}} \times \mathrm{g}' \, d\overset{\otimes}{D}_j \end{bmatrix} \tag{34}$$

Where,

$$Z_{11} \equiv \frac{\partial^2 \pi}{\partial Q_i^2} = R_{\mathrm{QQ}} - C_{\mathrm{QQ}} + a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{QQ}} - C_{\mathrm{QQ}}^{\max} \right] < 0,$$

$$Z_{12} \equiv \left[a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1 \right] \times C_{Q\overset{\otimes}{D}} \geq 0, \quad Z_{21} \equiv \left[a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1 \right] \times C_{\overset{\otimes}{D}Q} \geq 0,$$

$$Z_{22} \equiv \frac{\partial^2 \pi}{\partial \overset{\otimes}{D}_i^2} = C_{\overset{\otimes}{D}\overset{\otimes}{D}} \times \left[a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1 \right] - M_h'' < 0$$

and

$$\Gamma = Z_{11} \times Z_{22} - Z_{12} \times Z_{21}.$$

Comparative static derivative functions can be derived from (34):

$$\frac{\partial Q_i}{\partial m} = \frac{-(1 - v_i) \times Z_{12}}{\Gamma} < 0 \tag{35}$$

$$\frac{\partial Q_i}{\partial v_i} = \frac{m \times Z_{12}}{\Gamma} > 0 \tag{36}$$

$$\frac{\partial Q_i}{\partial a_u} = \frac{-Z_{22} \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max} \right] + Z_{12} \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times C_{\overset{\otimes}{D}}}{\Gamma} \tag{37}$$

$$\frac{\partial Q_i}{\partial \overset{\otimes}{D}_j} = \frac{a_u \times \mathrm{g}' \times \left[Z_{12} \times C_{\overset{\otimes}{D}} - Z_{22} \times (C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) \right]}{\Gamma} \tag{38}$$

$$\frac{\partial \overset{\otimes}{D}_i}{\partial m} = \frac{(1 - v_i) \times Z_{11}}{\Gamma} < 0 \tag{39}$$

$$\frac{\partial \overset{\otimes}{D}_i}{\partial v_i} = \frac{-m \times Z_{11}}{\Gamma} > 0 \tag{40}$$

$$\frac{\partial \overset{\otimes}{D}_i}{\partial a_u} = \frac{\mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[Z_{21} \times (C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) - Z_{11} \times C_{\overset{\otimes}{D}} \right]}{\Gamma} \tag{41}$$

$$\frac{\partial \overset{\otimes}{D}_i}{\partial \overset{\otimes}{D}_j} = \frac{a_u \times \mathrm{g}' \times \left[Z_{21}(C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) - Z_{11} \times C_{\overset{\otimes}{D}} \right]}{\Gamma} \tag{42}$$

Assuming that the conditions for economic stability are established ($\Gamma = Z_{11} \times Z_{22} - Z_{12} \times Z_{21} > 0$), the following economic implications can be inferred:

i) Equation (35) reveals that the air pollution tax levied by government decreases a firm's output capacity;
ii) Equation (39) shows that the air pollution tax levied by government reduces a firm's air pollution emissions;
iii) Equation (36) indicates that an increase in a firm's tax erosion rate increases its output capacity;
iv) Equation (40) demonstrates that raising a firm's tax erosion rate increases its air pollution emissions; and
v) All remaining factors may produce dissimilar results according to the signs of

the relevant variables, thus indicating no constancy in the results.

Proposition 1: Assuming that a firm is in a perfectly competitive market, the conditions for economic stability will be present when the financial policy instruments (e.g., air pollution tax and subsidy) are known. In addition, an increase in a firm's tax erosion rate will increase its output capacity [Equation (36)]; an increase in a firm's tax erosion rate will cause its air pollution emissions to rise [Equation (40)], and the air pollution tax levied by government decreases a firm's output capacity [Equation (35)].

4.3 Optimal Decisions for Government

In this section, the backward induction method is adopted to investigate how the government can formulate the optimal tax policy for controlling firms' air pollution emissions [10]. This entails the government setting the optimal air pollution tax rate and subsidy rate to maximize the benefits to society. These benefits include (i) consumer surplus, (ii) producer surplus, (iii) air pollution tax levied, and (iv) penalty income from firms whose air pollution emissions exceed the standards. However, the system also involves the negative items of social welfare, including (v) government expenditure because of subsidies granted to firms that engage in air pollution control, and (vi) the external cost ton the society due to because of air pollution emissions.

To simplify the calculation process, this paper defines the number of firms as *N*. Accordingly, the objective function for the maximization of the benefit to society can be expressed as follows:

$$\begin{aligned}\underset{a_u,\,m}{Max}\ S_W = & \sum_{i=1}^{N}\left[U(Q_i) - P\times Q_i\right] + \sum_{i=1}^{N}\left[R(Q_i) - C(Q_i, \overset{\otimes}{D}_i)\right] + \sum_{i=1}^{N} m\times(1-v_i)\times \overset{\otimes}{D}_i + \sum_{i=1}^{N} M_h(\overset{\otimes}{D}_i - D_0) \\ & - a_u \times \sum_{i=1, i\neq j}^{N} g_i(\overset{\otimes}{D}_j)\times\left[C_i(Q_i,\overset{\otimes}{D}_i) - C_i^{max}(Q_i,\overset{\exists}{D}_i)\right] - \sum_{i=1}^{N} I(\overset{\otimes}{D}_i)\end{aligned} \tag{43}$$

$$s.t.\quad \sum_{i=1}^{N} m\times(1-v_i)\times \overset{\otimes}{D}_i + \sum_{i=1}^{N} M_h(\overset{\otimes}{D}_i - D_0) = a_u \times \sum_{i=1, i\neq j}^{N} g_i(\overset{\otimes}{D}_j)\times\left[C_i(Q_i,\overset{\otimes}{D}_i) - C_i^{max}(Q_i,\overset{\exists}{D}_i)\right] \tag{44}$$

i) Where, $\sum_{i=1}^{N}\left[U(Q_i) - P\times Q_i\right]$ is the consumer surplus;

ii) $\sum_{i=1}^{N}\left[R(Q_i) - C(Q_i, \overset{\otimes}{D}_i)\right]$ is the producer surplus;

iii) $\sum_{i=1}^{N} m\times(1-v_i)\times \overset{\otimes}{D}_i$ is the air pollution tax levied;

iv) $\sum_{i=1}^{N} M_h(\overset{\otimes}{D}_i - D_0)$ is the penalty income from firms whose air pollution emissions exceed the standards;

v) $a_u \times \sum_{i=1, i\neq j}^{N} g_i(\overset{\otimes}{D}_j)\times\left[C_i(Q_i,\overset{\otimes}{D}_i) - C_i^{max}(Q_i,\overset{\exists}{D}_i)\right]$ is the government expenditure due to subsidies granted to firms that engage in air pollution control; and

vi) $\sum_{i=1}^{N} I(\overset{\otimes}{D}_i)$ is the external cost to society due to air pollution emissions.

If a government uses the air pollution tax exclusively to pay for the subsidy granted to firms that engage in air pollution, the objective function for the maximization of the benefit to society can be obtained by substituting (44) into (43), producing the following equation:]

$$\underset{a_u,\,m}{Max}\ S_W = \sum_{i=1}^{N}\left[U(Q_i) - P\times Q_i\right] + \sum_{i=1}^{N}\left[R(Q_i) - C(Q_i, \overset{\otimes}{D}_i)\right] - \sum_{i=1}^{N} I(\overset{\otimes}{D}_i) \tag{45}$$

5. SOLVING THE SUBGAME PERFECT EQUILIBRIUM BETWEEN FIRMS AND GOVERNMENT BY USING BACKWARD INDUCTION

5.1 Government's Optimal Subsidy Rate

By taking a first-order derivative of (45) with respect to the subsidy rate and making it equal to zero, the following can be obtained:

$$\left[(\mathrm{U}_{\mathrm{Q}} - P) + (\mathrm{R}_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}})\right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \mathrm{Q}_{\mathrm{i}}}{\partial a_u}\right) = (\mathrm{C}_{\overset{\otimes}{D}} + I') \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \overset{\otimes}{D}_i}{\partial a_u}\right) \tag{46}$$

By substituting $R_{\mathrm{Q}} - C_{\mathrm{Q}} = -a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max} \right]$ from (32) and $C_{\overset{\otimes}{D}} = \dfrac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1}$ from (33) into (46), the following is obtained:

$$\left[-a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times (C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) \right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \mathrm{Q}_{\mathrm{i}}}{\partial a_u}\right) = \left[\frac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1} + I' \right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \overset{\otimes}{D}_i}{\partial a_u}\right) \tag{47}$$

Assuming that the "rate of subsidy offered by the government to a firm engaging in air pollution control" and "firm's output capacity" are neutral, the optimal subsidy rate (an endogenous variable) to be offered by the government to firms that engage in air pollution control can be obtained using (47), as given by

$$a_u^* = \frac{\mathrm{I}' - (m \times (1 - v_i) + M_h')}{\mathrm{I}' \times \mathrm{g}_i(\overset{\otimes}{D}_j)} \tag{48}$$

5.2 Government's Optimal Air Pollution Tax Rate

To identify the optimal tax rate, a first-order derivative of (45) with respect to tax rate was taken and then equaled to zero, from which the following is obtained:

$$\left[(\mathrm{U}_{\mathrm{Q}} - P) + (\mathrm{R}_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}})\right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \mathrm{Q}_{\mathrm{i}}}{\partial m}\right) = (\mathrm{C}_{\overset{\otimes}{D}} + I') \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \overset{\otimes}{D}_i}{\partial m}\right) \tag{49}$$

Next, $R_{\mathrm{Q}} - C_{\mathrm{Q}} = -a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times \left[C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max} \right]$ (i.e., a firm's optimal first-order condition) from (32) and $C_{\overset{\otimes}{D}} = \dfrac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1}$ from (33) are substituted into (49):

$$\left[-a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times (C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) \right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \mathrm{Q}_{\mathrm{i}}}{\partial m}\right) = \left[\frac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1} + I' \right] \times \sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \overset{\otimes}{D}_i}{\partial m}\right)$$

$$\Rightarrow \left[-a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) \times (C_{\mathrm{Q}} - \mathrm{C}_{\mathrm{Q}}^{\max}) \right] \times \frac{\sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \mathrm{Q}_{\mathrm{i}}}{\partial m}\right)}{\sum_{i=1}^{\mathrm{N}} \left(\frac{\partial \overset{\otimes}{D}_i}{\partial m}\right)} = \left[\frac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1} + I' \right] \tag{50}$$

Assuming that "rate of subsidy offered by the government to a firm engaging in air pollution control" and "firm's output capacity" are neutral, (50) can be simplified to the following:

$$\frac{M_{\mathrm{h}}' + m \times (1 - v_i)}{a_u \times \mathrm{g}_i(\overset{\otimes}{D}_j) - 1} = -I'$$

Accordingly, the optimal tax rate (an endogenous variable) to be levied by the government can be obtained from the following:

$$m^* = \frac{I' \times \left[1 - a_u \times g_i(\overset{\otimes}{D}_j)\right] - M'_h}{(1 - v_i)} \quad (51)$$

By considering the spillover effect, the optimal air pollution tax rate and optimal subsidy rate models can be inferred and used to verify the following proposition:

Proposition 2: Assuming that the government's subsidy rate (a_u) for firms that engage in air pollution control is an exogenous variable, the stronger the spillover effect (i.e., the lower $g_i(\overset{\otimes}{D}_j)$ is), the greater the tax rate should be. In addition, when representative firms' tax erosion rate (v_i) increases, this is a signal to government that it should increase its tax rate to encourage the firms to research and develop air pollution control technologies, which will subsequently increase the spillover effect and enhance the benefit to society (S_W).

6. CONCLUSION

This study investigated game analyses for tax evaders and tax-levying governments. In the two-stage game settings, when all market participants play the tax game while knowing all information, backward induction can be used to solve the subgame perfect equilibrium between firms and government. The calculation process is divided into two stages; the identification by firms of the optimal production output and air pollution emissions, and the substitution of firms' production output and air pollution emissions into the social welfare function by government to determine the optimal tax and subsidy rates to be offered to firms engaging in air pollution control. Assuming that firms are in a perfectly competitive market and that financial policy instruments (i.e., air pollution tax and subsidy rate) are known, conditions for economic stability can be established. The present study also demonstrated that an increase in a firm's tax erosion rate increases its output capacity and air pollution emissions, whereas an increase in the air pollution tax levied by government decreases a firm's output capacity.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Allingham MG, Sandmo A. Income tax evasion: A Theoretical analysis. Journal of Public Economics. 1972;1(3):323-338.
2. Wang LFS, Conant JL. Corporate tax evasion and output decisions of the uncertain monopolist. National Tax Journal. 1988;41(4):579-581.
3. Yaniv G. A note on the tax evading firm. National Tax Journal. 1995;48:113-120.
4. Baron DP. Private politics, corporate social responsibility and integrated strategy. Journal of Economics and Management Strategy. 2001;10(1):7-45.
5. Wu TC, Yen CT. Tax evasion and monopoly output decisions in the presence of corporate social, Agriculture and Economics. 2011;46:85-101.
6. Hsu YC, Tsui SW. Environmental policies with financing emission tax under spillover effect, Journal of Agricultural Economics. 2005;78:151-183.
7. Slemrod, Joel, and Yitzhaki, Shlomo. Tax avoidance, evasion, and administration. National Bureau of Economic Research Working Paper. 2000;7473:1-76.
8. Yitzhaki S. On the excess burden of tax evasion. Public Finance Review. 1987; 15(2):123-137.
9. Harford JD. Firm behavior under imperfectly enforceable pollution standards and taxes. Journal of Environmental Economics and Management. 1978; 5(26):26-43.
10. Khanna M. Non-Mandatory approaches to environmental protection. Journal of Economic Surveys. 2002;15(3):291-324.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history/26298