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Unsteady Oscillatory Couette Flow between Vertical Parallel Plates with Constant Radiative Heat Flux

K. W. Bunonyo[1*], E. Amos[2] and I. C. Eli[1]

[1]*Department of Mathematics and Statistics, Federal University, Otuoke, Yenagoa, Nigeria.*
[2]*Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.*

Authors' contributions

This research work was carried out in collaboration between all authors. Authors KWB and EA designed the study, performed the mathematical formulation and wrote the first draft of the manuscript. Authors KWB, EA and ICE managed the analyses of the study. All the authors managed the literature searches, read and approved the final manuscript.

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Abstract

In the present study, Unsteady Oscillatory Couette Flow between vertical parallel plates with constant radiative heat flux is considered. The mathematically formulated governing equations are simplified with dimensionless variables and coupled ordinary differential equations were obtained. Rosseland approximation is used and the present study is therefore for a case of to an optically thick fluid like blood. The transformed set of coupled nonlinear ordinary differential equations is then solved analytically using the perturbation method, and the velocity and temperature functions are simulated. It is observed that increasing the thermal radiation parameter Rd increases the blood flow, in addition, the oscillatory frequency ω also improves the flow profiles such as temperature $\theta\left(y\right)$ and velocity $w(y)$.

Keywords: Oscillatory; Couette heat flux; blood.

**Corresponding author: E-mail: wilcoxonnetb@hotmail.com, wilcoxbk@fuotuoke.edu.ng;*

1 Introduction

It is found that oscillatory flow results in higher rates of heat transfer between parallel plates. The flow between parallel plate has extensive application in MHD power generators, petroleum industry, purification of crude oil, pumps accelerators, polymers technology and in several material processing applications such as metal forming, continuous casting, wire and glass fibre drawing, and others [1,2]. More study has been done in the case of horizontal parallel plates that vertical parallel plate. Schlichting and Gersten presented an analysis of flow formation in Couette motion between two vertical parallel plates [3]. Convection in vertical channels has been studied extensively under different physical effects due to its importance in many engineering applications such as the construction of passive solar systems for energy conversion, cooling of nuclear reactors, construction of heat exchangers, cooling of electronic equipment, geothermal systems [4–15]. However, there are few authors who studied convection in unsteady Couette motion between two parallel vertical plates. Singh [16] studied the effect of convection in unsteady Couette motion between two vertical parallel plates. This problem was further extended in the case of MHD by Jha [17]. Jain and Gupta [18] studied fully developed laminar free convection Couette flow between two parallel vertical plates with a transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate whose motion was uniform. Barletta and Magyari [19] investigate Couette flow of a Bingham fluid in a vertical parallel plane channel with fixed temperature differential across the walls. In their investigation, they consider the physical effect of external shear in the form of fluid motion. Barletta et al. [20] studied fully-developed combined force, and free convection Couette flow with viscous dissipation in a vertical channel. Daniel et al. [21] investigated the steady two-dimensional electrical magnetohydrodynamic (MHD) nanofluid flow over a stretching/shrinking sheet. The effects of stretching and shrinking parameter, as well as electric and magnetic fields, thermal radiation, viscous and Joule heating in the presence of slip, heat and mass convection boundary conditions at the surface, are imposed and studied.

Bunonyo et al. [22] studied Blood Flow through Stenosed Artery with Heat in the Presence of Magnetic Field. In the study, they convert the coupled partial differential equation into an ordinary differential equation using the oscillatory perturbation condition and are solved using Frobenius method and observed that some of the physical parameters affect the flow profiles. Bunonyo et al. [23] studied blood flow through an indented artery and assumed blood to be Jeffrey fluid. They formulated governing equations are transformed coupled Bessel differential equation and solved analytically. The effects of various physical parameters such as Prandtl number for blood as 21.

The main goal of the present research paper is to study the unsteady oscillatory Couette flow between vertical parallel plates, where the moving plate is subjected to constant radiative heat flux and the plate at rest is isothermal.

2 Mathematical Formulation

Consider the unsteady oscillatory, conducting, incompressible, viscous and radiating fluid flow between two vertical parallel plates separated by a distance R_0. The x'-axis is taken along one of the plates in the vertically upward direction and the y'- axis is taken normal to the plate. Under the assumption, the governing equations for the unsteady oscillatory Couette fluid flow through a parallel plate with Boussinesq approximation may be written as:

Momentum Equation

$$\rho \frac{\partial w'}{\partial t'} = \rho g \beta \left(T' - T'_w\right) - \frac{\partial P'}{\partial x'} + \mu \frac{\partial^2 w'}{\partial y'^2} \tag{1.1}$$

Energy Equation

$$\rho C p \frac{\partial T'}{\partial t'} = k_T \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{1.2}$$

The corresponding boundary conditions are as follows

$$\begin{aligned} & w' = w_\infty, \frac{\partial T'}{\partial y'} = -\frac{q_r}{k_T} \quad \text{at} \quad y' = 0 \\ & w' = 0, T' = T'_w \quad \text{at} \quad y' = R_0 \end{aligned} \tag{1.3}$$

The radiative heat flux term is simplified by making use of the Rosseland approximation [24] as

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{1.4}$$

Where σ is Stefan-Boltzmann constant k^* is the mean absorption coefficient. We know that by using the Rosseland approximation we limit our analysis to optically thick fluids like blood. If temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature, then the Taylor series for T'^4 about T'_w, after neglecting the higher order terms, is given by:

$$T'^4 \cong 4T'^3_w T' - 3T'^4_w \tag{1.5}$$

We can emphasise here that equation (1.5) is widely used in computational fluid dynamics involving radiation absorption problems [25] in expressing the terms T'^4 as a linear function. In view of Equation (1.4) and (1.5), Equation (1.2) can be reduced to

$$\rho Cp \frac{\partial T'}{\partial t'} = k_T \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T'^3_w}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \tag{1.6}$$

Equation (1.1) and (1.5) are coupled differential equation. To solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities as:

$$\begin{aligned} & y = \frac{y'}{R_0}, t = \frac{t'\nu}{R_0^2}, w = \frac{w'}{w_\infty}, \theta = \frac{T' - T'_w}{\left(q_r / k_T\right)}, \\ & Pr = \frac{\mu Cp}{k_T}, Rd = \frac{k_T k^*}{4\sigma T'^3_w}, Gr = \frac{g\beta R_0^2 q_r}{w_\infty k_T \nu} \end{aligned} \tag{1.7}$$

Where Gr is thermal Grashof number, Pr is the Prandtl number, Rd is the radiation parameter, t is the dimensionless time, w is the dimensionless velocity, y is the dimensionless coordinate axis normal to the wall, μ is the dynamic viscosity and θ is the dimensionless temperature.

Using the dimensionless quantities in Equation (1.7) into Equations (1.6) and (1.1) the Governing Equations can be reduced to the following dimensionless form as:

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial y^2} + Gr\theta \tag{1.8}$$

$$3RdPr\frac{\partial \theta}{\partial t} = \left(3Rd + 4\right)\frac{\partial^2 \theta}{\partial y^2} \tag{1.9}$$

The corresponding dimensionless boundary conditions are as:

$$w = 1, \frac{\partial \theta}{\partial y} = -1 \quad \text{at} \quad y = 0$$
$$w = 0, \theta = 0 \quad \text{at} \quad y = 1 \tag{1.10}$$

2.1 Method of solution

We considered blood as the fluid and it flows through the vertical plates is purely oscillatory, thus, equations (1.8) & (1.9) solved after considering the following perturbation involving the velocity and temperature as:

$$\left.\begin{aligned} w &= w_0 e^{i\omega t} \\ \theta &= \theta_0 e^{i\omega t} \end{aligned}\right\} \tag{1.11}$$

Putting equation (1.11) into equations (1.8) and (1.9), we obtained the oscillatory governing equations as:

$$\frac{\partial^2 w_0}{\partial y^2} - w_0 i\omega = -Gr\theta_0 \tag{1.12}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} - \phi_1 \theta_0 = 0 \tag{1.13}$$

where $\phi_1 = \left(\frac{3RdPri\omega}{(3Rd + 4)} \right)$

The corresponding boundary conditions in perturbed form are as follows:

$$w_0 = e^{-i\omega t}, \frac{\partial \theta_0}{\partial y} = -e^{-i\omega t} \quad \text{at} \quad y = 0$$

$$w_0 = 0, \theta_0 = 0 \quad \text{at} \quad y = 1 \tag{1.14}$$

Equation (1.13) is solved and the following general solution as:

$$\theta_0 = A\cos h(\phi_1 y) + B\sin h(\phi_1 y) \tag{1.15}$$

We can now use the boundary conditions in Equation (1.14) and solve for the constants A and B in Equation (1.15), we obtain the following:

$$\theta_0 = \frac{e^{-i\omega t}}{\phi_1}\left(tanh(\phi_1)\cos h(\phi_1 y) - \sin h(\phi_1 y) \right) \tag{1.16}$$

Putting Equation (1.16) into the oscillatory dimensionless temperature, we have the perturbed temperature equation to be

$$\theta = \frac{1}{\phi_1}\left(tanh(\phi_1)\cos h(\phi_1 y) - \sin h(\phi_1 y) \right) \tag{1.17}$$

where $A = \frac{e^{-i\omega t}}{\phi_1}\frac{\sin h(\phi_1)}{\cos h(\phi_1)} = \frac{e^{-i\omega t}}{\phi_1}tanh(\phi_1),\ B = -\frac{e^{-i\omega t}}{\phi_1}$

Putting Equation (1.16) into Equation (1.12), it becomes the following inhomogeneous ordinary differential Equation as follows:

$$\frac{\partial^2 w_0}{\partial y^2} - w_0 i\omega = \frac{e^{-i\omega t}Gr}{\phi_1}\left(\sin h(\phi_1 y) - tanh(\phi_1)\cos h(\phi_1 y)\right) \tag{1.18}$$

The solution to the homogeneous part of Equation (1.18) is obtained as:

$$w_{oh} = A_1 cos\left(\sqrt{\omega}y\right) + B_1 sin\left(\sqrt{\omega}y\right) \tag{1.19}$$

And the particular solution is given as:

$$w_{op} = A_2 cosh(\phi_1 y) + B_2 sinh(\phi_1 y) \tag{1.20}$$

We differentiate Equation (1.20) and substitutes it into Equation (1.18) and solve, we obtain the general solution as follows:

$$w_0 = A_1 cos\left(\sqrt{\omega}y\right) + B_1 sin\left(\sqrt{\omega}y\right) + A_2 cosh(\phi_1 y) + B_2 sinh(\phi_1 y) \tag{1.21}$$

where $A_2 = \frac{e^{-i\omega t}Gr}{\phi_1}\frac{tanh(\phi_1)}{\left(Ai\omega - \phi_1^2\right)}, B_2 = \frac{e^{-i\omega t}Gr}{\phi_1\left(\phi_1^2 - i\omega s\right)}$

To resolve for the other constants in Equation (1.21) we have to make use of the boundary conditions in Equation (1.14) as follows:

$$w_0 = A_1 cos\left(\sqrt{\omega}y\right) + B_1 sin\left(\sqrt{\omega}y\right) + A_2 cosh(\phi_1 y) + B_2 sinh(\phi_1 y) \tag{1.22}$$

So that $w = \left(A_1 cos\left(\sqrt{\omega}y\right) + B_1 sin\left(\sqrt{\omega}y\right) + A_2 cosh(\phi_1 y) + B_2 sinh(\phi_1 y)\right)e^{-i\omega t}$ (1.23)

Where

$$B_1 = 1 - \frac{\left[\phi_2 + \phi_3 + \left\{\frac{e^{-i\omega t}Gr}{\phi_1}\frac{tanh(\phi_1)}{\left(i\omega - \phi_1^2\right)}\right\}cosh(\phi_1)\right]}{sin\left(\sqrt{\omega}\right)},\quad A_1 = \left\{\frac{e^{-i\omega t}Gr}{\phi_1}\frac{tanh(\phi_1)}{\left(\phi_1^2 - Ai\omega\right)} + e^{-i\omega t}\right\},\quad \phi_2 = \left\{\frac{e^{-i\omega t}Gr}{\phi_1\left(\phi_1^2 - i\omega s\right)}sinh(\phi_1)\right\},$$

$$\phi_3 = \left\{\frac{e^{-i\omega t}Gr}{\phi_1}\frac{tanh(\phi_1)}{\left(\phi_1^2 - Ai\omega\right)} + e^{-i\omega t}\right\}cos\left(\sqrt{\omega}\right)$$

3 Results

In order to get physical insight into the problem, numerical simulation was carried out for the temperature $\theta(y)$ and the velocity $w(y)$ profiles and we vary the values of some of the pertinent parameters such as

Radiation parameter Rd, Grashof number Gr, Prandtl number Pr, and periodic time t on flow profiles. The results are presented for temperature and velocity profiles follow:

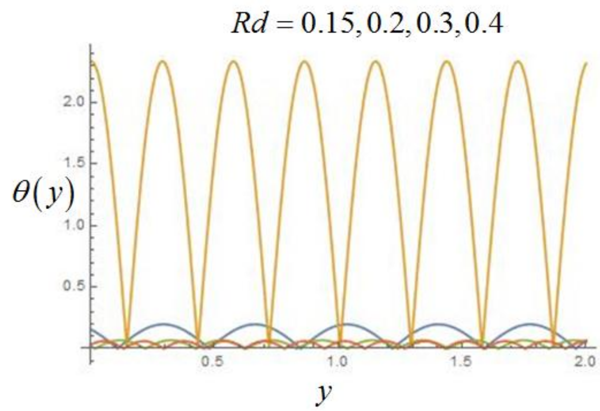

Fig. 1. Influence of radiation on temperature while other parameters are unchanged

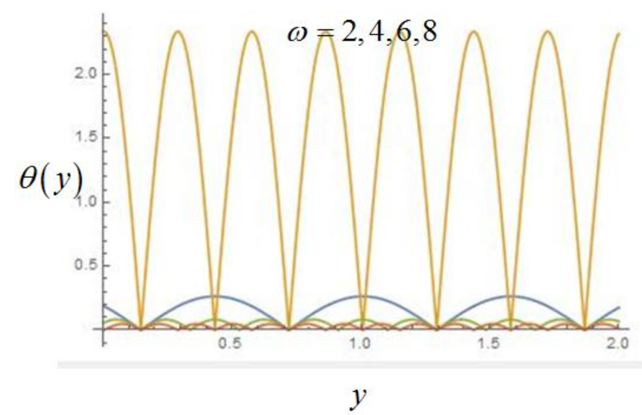

Fig. 2. Influence of frequency ω on temperature while other parameters are unchanged

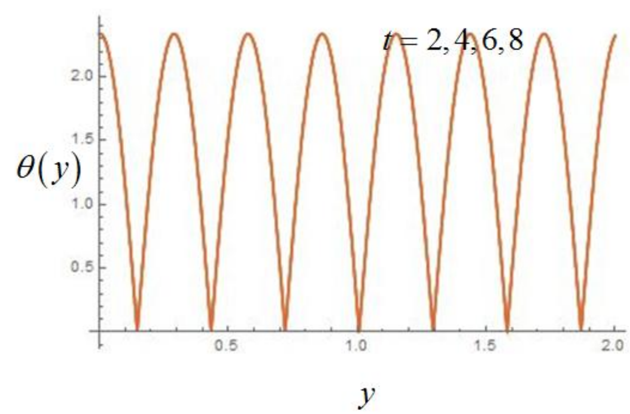

Fig. 3. Influence of t on temperature while other parameters are unchanged

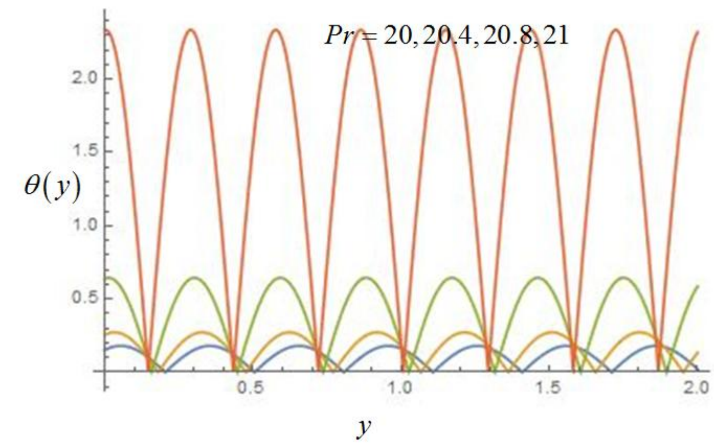

Fig. 4. Influence of Prandtl's number on temperature while other parameters are unchanged

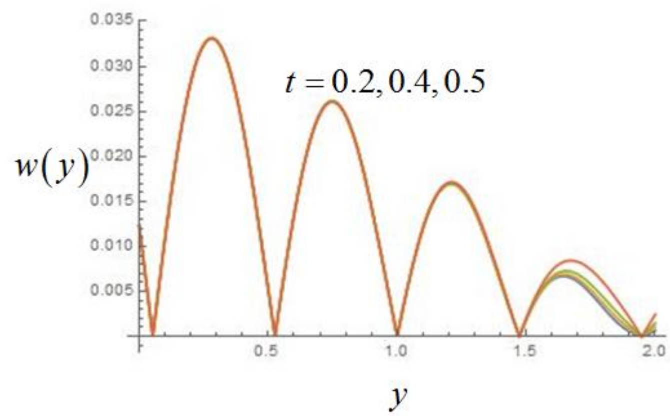

Fig. 5. Influence of t on velocity profile while other parameters are unchanged

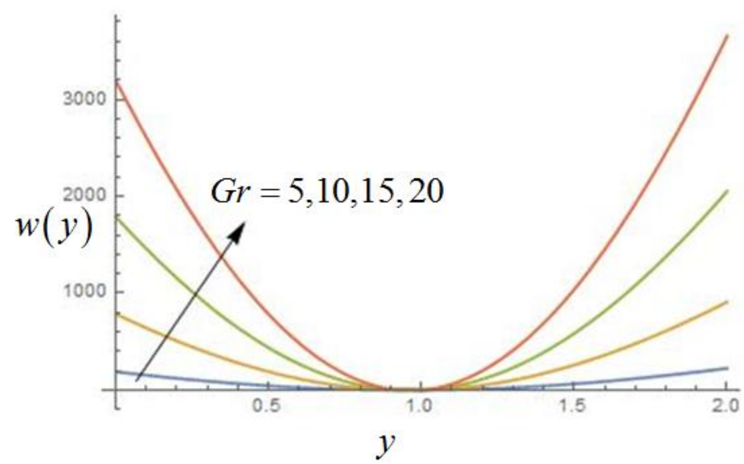

Fig. 6. Influence of Grashof Gr velocity profile while other parameters remain unchanged

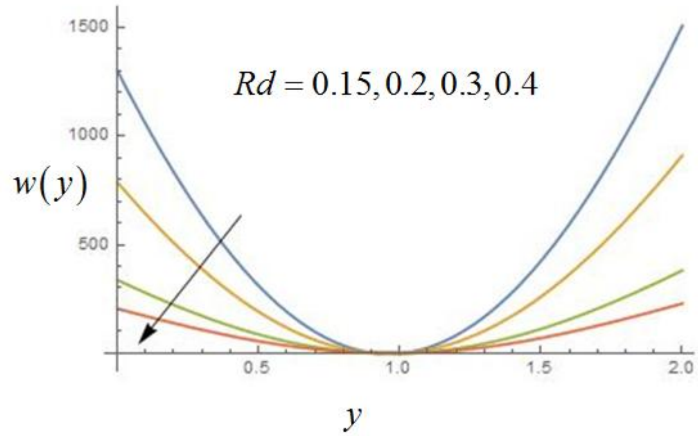

Fig. 7. Influence of radiation Rd on velocity profile other parameters are unchanged

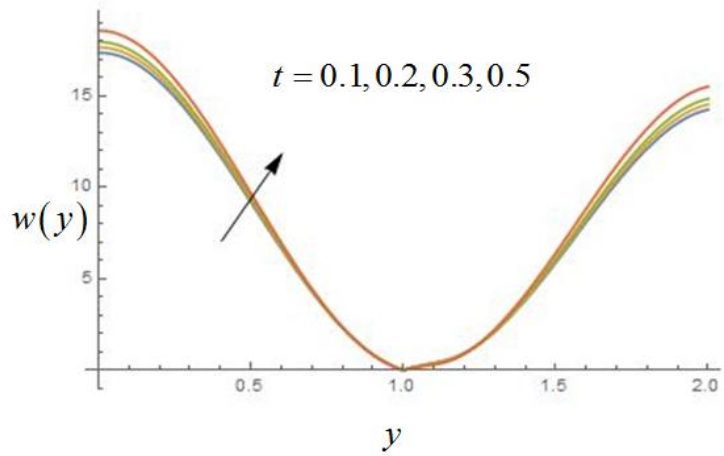

Fig. 8. Influence of time t on the velocity profile other parameters are unchanged

4 Discussion

It is observed in Fig. 1 that the temperature profile is caused to oscillate as the radiation is increased, but seem to reduce in amplitude with radiation parameter $Rd = 0.15, 0.2, 0.3, 0.4$ while other pertinent parameters remained unchanged.

Fig. 2 shows that as the oscillatory parameter $\omega = 2, 4, 6, 8$ increases the temperature profile $\theta(y)$ is observed to also oscillate in the channel of the fluid flow thereby causing the temperature to rise above its normal level. In addition, we observed in Fig. 3 that the time increase doesn't really have any relative effect on the temperature profile because it remained steady as the value of the temperature and other pertinent parameters remain unchanged.

In Fig. 4, we observed that as the blood lost some of its characteristics in Prandtl number Pr , the temperature profile $\theta(y)$ still oscillate in the channel as such it the amplitude of oscillation equally reduced as the other regular parameters are kept constant. However, we can see in Fig. 5 that there is clear drop in oscillation over time, that is, as the stream is kept for long we observed the drop in velocity profile $w(y)$, which clearly means, for us to achieve our desired results the flow must be observe and see if it require some sort of boost. If the velocity profile is allowed to continuously reduce its amplitude it could cause than good.

It is obvious in Fig. 6 that as the Grashof number $Gr = 5,10,15,20$ increases the flow velocity is caused to increase as well, thus the Grashof number Gr helps the oscillatory behaviour of the fluid. Fig. 7 depicts that as the radiation $Rd = 0.15, 0.2, 0.3, 0.4$ increases, the flow velocity $w(y)$ tends to decrease to a point of stability then began to increase, thus, it simply means that if the radiation rate is well controlled it will be helpful to improve the oscillatory flow through a channel. Also, Fig. 8 depicts that as the time value is increased, $t = 0.1, 0.2, 0.3, 0.5$ the velocity profile $w(y)$ continues to improve, meaning if we keep all other parameters constant the flow can stream on, and that would help improve life.

5 Conclusion

After carrying out the theoretical study of an unsteady oscillatory Couette flow in a channel, we can conclude with the following findings:

1. The temperature profile $\theta(y)$ was adversely affected by radiation Rd while other pertinent parameters are kept constant. It clearly shows that the radiation caused the fluid to oscillate in the channel.
2. We equally conclude that the oscillatory parameter ω s played a greater role in causing the fluid to oscillate inside the Couette channel while other important parameters are kept constant on the temperature profile $\theta(y)$.
3. We can also conclude that the time parameter plays less or negligible role in the flow process because the blood leaves the left ventricle with an organ required temperature to be supplied to all sensitive parts since the pump action of the heart pulsate
4. The Grashof number Gr affects the velocity profile $w(y)$ while other parameters remain unchanged.
5. It is of a vital point to conclude that the radiation parameter Rd also affects the velocity profile while other important parameters are kept constant in an oscillatory Couette flow in a parallel vertical channel.
6. Finally, the time of flow played a good role in our simulated results above such that the longer the stream is kept to observe we can say that it affects the velocity profile $w(y)$.

Competing Interests

Authors have declared that no competing interests exist.

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